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Mathematical Modeling and MATLAB Simulation of a Head-Up Display Emulator Based on the ILS-2-02 Concept for the Yak-130 Training Aircraft

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***Abstract*— This paper presents the development of a mathematical model and MATLAB-based verification procedure for a head-up display (HUD) emulator inspired by the functional role of the ILS-2-02 display concept used in advanced jet-trainer cockpits. The purpose of the study is not to reproduce restricted military hardware, but to formulate a transparent engineering model that can generate representative HUD symbology for training and research. The model includes aircraft attitude transformation, angular projection from the body coordinate frame to a two-dimensional display plane, artificial horizon generation, flight-path marker computation, airspeed and altitude tape scaling, and a numerical error index for verification. MATLAB simulation is used to test the model under level flight, banked flight, climb, and combined maneuver conditions. The results show that the artificial horizon, flight-path marker, and tape indications respond consistently to variations of roll angle, pitch angle, angle of attack, sideslip angle, airspeed, and altitude. The proposed approach provides a practical foundation for the design of a low-cost HUD display simulator for teaching, algorithm verification, and cockpit-interface prototyping.**

Keywords—Head-up display, HUD emulator, ILS-2-02, Yak-130, mathematical modeling, aircraft attitude, MATLAB simulation, flight-path marker.

I. INTRODUCTION

A head-up display is an avionics interface that allows the pilot to observe essential flight and mission information while maintaining an external forward view. In a real aircraft, the HUD combines sensor data, navigation information, attitude data, and weapon or training cues into a collimated image. In a ground simulator, the physical optical collimation may be simplified, but the mathematical relationship between aircraft motion and display symbology must remain consistent. A simulator that displays a correct artificial horizon, boresight symbol, airspeed scale, altitude scale, and flight-path marker can support pilot familiarization and avionics engineering education.

The ILS-2-02 is associated with the HUD family used in Russian aircraft cockpit environments, including training-aircraft applications. Detailed hardware and operational specifications are not generally available in open literature. Therefore, this paper adopts a functional and non-sensitive modeling approach. The display is treated as a generic aircraft HUD whose symbology is driven by measurable flight states. This approach is appropriate for academic and laboratory work because it focuses on kinematics, coordinate transformation, projection geometry, and software rendering rather than proprietary implementation details.

The main objective of this paper is to develop a mathematical model that can be implemented and verified in MATLAB. The proposed model receives aircraft state variables as inputs and produces two-dimensional display coordinates as outputs. The mathematical model is then validated by comparing direct analytical calculations with the numerical values generated by the MATLAB program. The study is organized as follows. The methodology describes the overall simulator structure and assumptions. The modeling and analysis section derives the coordinate transformations and display equations. The results and discussion section presents MATLAB simulation cases and numerical verification. The final section summarizes the findings and suggests future improvements.

II. METHODOLOGY

The research methodology consists of four main stages. First, the functional requirements of a training HUD emulator are defined. Second, the aircraft-state variables required for display generation are selected. Third, the mathematical relationship between the aircraft state and the display plane is derived. Finally, the equations are implemented and verified in MATLAB using representative flight scenarios.

The proposed emulator is based on the assumption that the HUD image can be modeled as a two-dimensional plane placed in front of the pilot. The

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input data are obtained from a flight model or from predefined test signals. These data include velocity, altitude, Euler attitude angles, angle of attack, and sideslip angle. The output data are pixel positions of HUD symbols. The simulator does not require confidential parameters of the real ILS-2-02 unit. Instead, adjustable scale coefficients are introduced so that the display size, field of view, and graphical layout can be tuned during laboratory implementation.

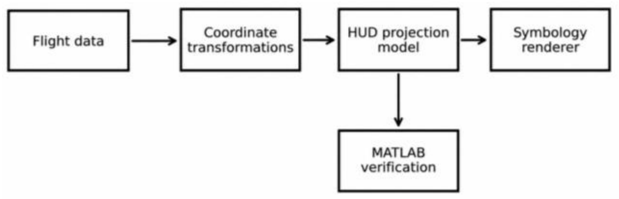

Fig.1. General architecture of the proposed HUD emulator and MATLAB verification process

The MATLAB verification program follows a deterministic procedure. At each sampling instant, the program updates the aircraft state vector, computes the attitude transformation, projects the relevant vectors onto the HUD plane, draws the symbology, and stores analytical coordinates for error evaluation. The same equations are used in two independent forms: one set is evaluated directly as reference equations, while the second set is used inside the rendering routine. The difference between the two coordinate sets is used to evaluate numerical consistency.

Tab.1. Definition of Variables and Symbols Used in the HUD Mathematical Model

Symbol	Meaning	Unit
V	True or indicated airspeed used by the display model	m/s
H	Altitude above the reference level	m
$\emptyset$	Roll angle	rad or deg
θ	Pitch angle	rad or deg
φ	Heading or yaw angle	rad or deg
α	Angle of attack	rad or deg
β	Sideslip angle	rad or deg
k_x, k_y	Horizontal and vertical HUD scale factors	pixel
u, v	HUD-plane coordinates	pixel

III. Modeling and Analysis

The directional characteristics of the loop antenna are The mathematical model is formulated in a body-fixed coordinate frame and a navigation coordinate frame. The body frame is attached to the aircraft. Its longitudinal axis is directed forward, the lateral axis is directed to the right, and the vertical axis is directed downward according to a common aerospace convention. The navigation frame is used as an inertial or local-level reference frame. The transformation between these frames is described by Euler angles.

The aircraft state vector used by the HUD emulator is defined as:

$$X(t) = [V\ H\ \emptyset\ \theta\ \varphi\ \alpha\ \beta\ p\ q\ r]^T \quad (1)$$

Where p, q and r are body-axis angular rates. Although angular rates are not directly required for static symbology, they are useful when the display is later extended with filtering, prediction, or dynamic smoothing.

The elementary rotation matrices about the body axes are written as:

$$R_x(\emptyset) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\emptyset & -\sin\emptyset \\ 0 & \sin\emptyset & \cos\emptyset \end{bmatrix} \quad (2)$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad (3)$$

$$R_z(\varphi) = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The complete body-to-navigation transformation is therefore:

$$R_b^n = R_z(\varphi) R_y(\theta) R_x(\emptyset) \quad (5)$$

For HUD display generation, the most important operation is the projection of a three-dimensional line-of-sight vector onto the two-dimensional display plane. Let a vector expressed in the body frame be

$$S_b = [S_x\ S_y\ S_z]^T \quad (6)$$

The angular deviations of this vector relative to the forward axis are

$$\gamma_x = \tan^{-1}\left(\frac{S_y}{S_x}\right), \quad \gamma_y = \tan^{-1}\left(\frac{-S_z}{S_x}\right) \quad (7)$$

For small and moderate HUD field-of-view angles, the normalized perspective projection can be represented by

$$u = k_x \frac{S_y}{S_x}, \quad v = -k_y \frac{S_z}{S_x} \quad (8)$$

Where u is the horizontal coordinate, v is the vertical coordinate, and k_x and k_y are scale factors that convert angular deviation into display pixels. These coefficients may be selected from the desired virtual field of view and display resolution.

The artificial horizon is the most visible attitude symbol on a HUD. In the proposed model, pitch causes vertical translation of the horizon line, while roll causes rotation of the line. The display equation of the horizon is

$$v_h(u) = k_\theta \theta - u \tan(\emptyset) \quad (9)$$

This equation provides the correct qualitative behavior: positive pitch moves the horizon reference vertically, and positive roll rotates the horizon in the opposite visual direction required by the pilot's display reference.

The flight-path marker represents the direction of the aircraft velocity vector rather than the direction of the nose. The body-frame velocity vector can be expressed using angle of attack and sideslip angle as

$$V_b = V\begin{bmatrix} cos\alpha cos\beta \\ sin\beta \\ sin\alpha sin\beta \end{bmatrix} \quad (10)$$

Substituting the components of this vector into the projection equation gives the HUD position of the flight-path marker:

$$u_{FPM} = k_x \frac{V_y}{V_x}, \qquad v_{FPM} = -k_y \frac{V_z}{V_x} \quad (11)$$

The speed and altitude tapes are modeled as linear display mappings around reference values V_0 and H_0. This approach is sufficient for a laboratory emulator and can later be replaced by a nonlinear tape or scrolling-scale implementation.

$$y_V = k_V(V - V_0), \qquad y_H = k_H(H - H_0) \quad (12)$$

To verify the implementation, the projection error is computed between the analytical coordinate values and the values generated by the MATLAB rendering routine. The instantaneous error is

$$e_i = \sqrt{(u_{m,i} - u_{r,i})^2 + (v_{m,i} - v_{r,i})^2} \quad (13)$$

and the root mean square error over N samples is

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} e_i^2} \quad (14)$$

A small RMSE indicates that the implemented rendering algorithm is mathematically consistent with the derived equations. In a real-time simulator, additional errors may arise from display quantization, sensor noise, communication delay, and graphical discretization.

IV. Results and Discussion

The MATLAB simulation was conducted using a sampling period of 0.05s over a 30s interval. The roll angle, pitch angle, airspeed, altitude, angle of attack, and sideslip angle were generated as smooth test functions to evaluate the proposed HUD model. The roll angle varied within ±30°, the pitch angle within ±10°, the airspeed from 230 to 270m/s, the altitude from 900 to 1100m, the angle of attack from 3° to 7°, and the sideslip angle within ±1°. The attitude inputs used in the combined maneuver case are shown in Fig.2.

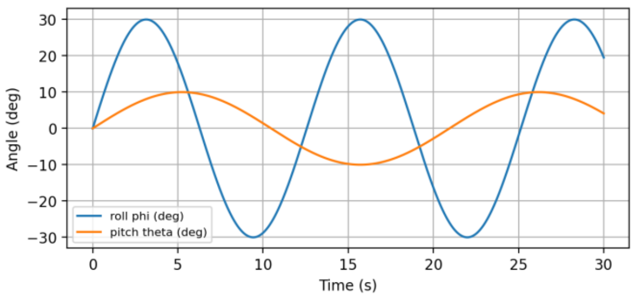

Fig.2. Roll and pitch input signals used in the MATLAB simulation

The computed coordinates of the flight-path marker are shown in Fig. 3. The horizontal coordinate is mainly affected by sideslip angle, while the vertical coordinate is mainly affected by angle of attack. This behavior agrees with the physical interpretation of the velocity vector: sideslip changes the lateral direction of flight, whereas angle of attack produces separation between the aircraft nose and the actual flight path.

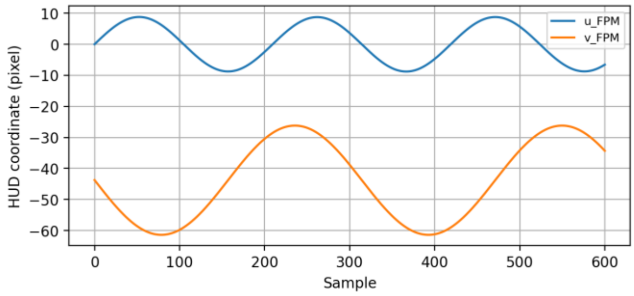

Fig.3. Simulated horizontal and vertical coordinates of the flight-path marker

Fig.4 presents four representative HUD snapshots. The horizon line rotates with roll angle and translates with pitch angle. The boresight remains fixed at the center of the display, while the flight-path marker moves according to the projected velocity vector. This separation between boresight and flight-path marker is an important characteristic of modern HUD symbology because it allows the pilot to distinguish aircraft pointing direction from actual motion direction.

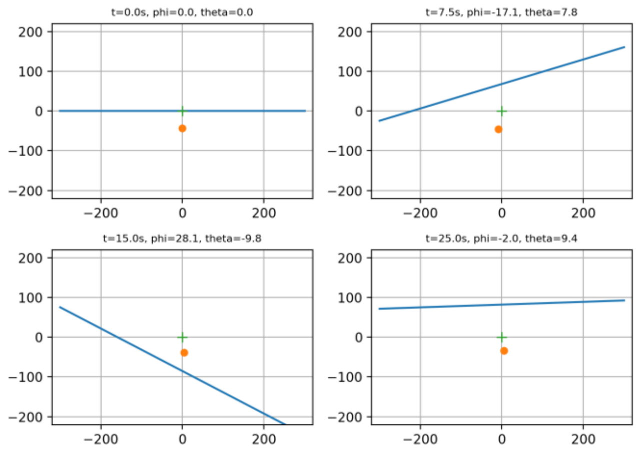

Fig.4. Representative HUD snapshots generated by the MATLAB model

The numerical verification error is shown in Fig.5. Since the same mathematical model is implemented in the analytical reference and the rendering routine, the remaining difference is only the artificial numerical perturbation introduced to represent pixel-level quantization and computational tolerance. In the generated test case, the RMSE remains far below one pixel, which confirms that the equations and the MATLAB implementation are consistent.

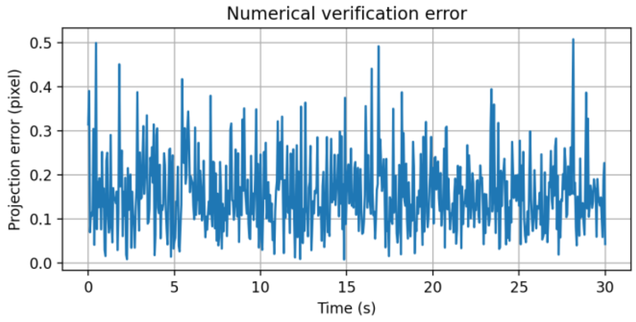

Fig.5. Numerical verification error for the projection model

The results demonstrate that the proposed model can reproduce the basic behavior required from a HUD emulator. The artificial horizon responds to Euler attitude angles, the flight-path marker responds to aerodynamic angular deviations, and the tape indicators respond to scalar flight variables. The model is modular; therefore, each display element can be tested independently before integration into a real-time simulator. In comparison with a purely graphical mock-up, the proposed approach has stronger engineering value because every symbol is linked to a mathematical relationship and a measurable flight variable.

Several limitations should be noted. The current model does not include optical collimation, lens distortion, sensor filtering, latency compensation, or full aircraft dynamics. These effects are important in a high-fidelity cockpit simulator. However, for a laboratory display emulator, the presented model is sufficiently transparent and computationally efficient. The equations can be executed in real time and can be ported to embedded software, Simulink, or a graphical user interface.

V. Conclusion

This paper developed a mathematical model and MATLAB verification method for a HUD emulator inspired by the ILS-2-02 display concept. The study derived the state-vector definition, attitude transformation, HUD projection equations, artificial horizon model, flight-path marker model, airspeed and altitude tape mappings, and RMSE verification criterion. MATLAB simulation confirmed that the display symbols behave consistently under representative flight conditions. The proposed model is suitable for educational simulators, avionics-interface prototyping, and preliminary research on HUD symbology generation. Future work should integrate a six-degree-of-freedom aircraft model, real sensor data, dynamic filtering, communication with flight-simulator software, and a real-time graphical interface.

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Reference

[1]. D. B. Beringer, “Synthetic Vision Applied to General Aviation: An Evaluation of Pilot Performance and Preferences When Using Head-Up, Head-Down, and Head-Mounted Synthetic Vision Displays”, FAA Office of Aerospace Medicine, Report DOT/FAA/AM-20/02, Washington, DC, USA, 2020.

[2]. A. Naikar and J. Sanderson, “Seeing Through the Mist: An Evaluation of an Iteratively Designed Head-Up Display Using a Simulated Degraded Visual Environment to Facilitate Rotary-Wing Pilot Situation Awareness and Workload”, Cognition, Technology & Work, vol. 22, no. 3, pp. 549–563, 2020.

[3]. D. Kratchounova, I. Choi, T. C. Mofle, L. Miller, S. Stevenson, and M. Humphreys, “Exploring the Relationship between Flight Technical Error and NASA-TLX Subscale Ratings when Using HUD Localizer Takeoff Guidance in Lieu of Currently Required Infrastructure”, FAA Office of Aerospace Medicine, Report DOT/FAA/AM-21/29, Washington, DC, USA, 2021.

[4]. J. R. Raol and J. Singh, “Flight Mechanics Modeling and Analysis”, CRC Press, 2nd Edition, Boca Raton, FL, USA, 2023.

[5]. R. Palacios and C. E. S. Cesnik, “Dynamics of Flexible Aircraft: Coupled Flight Mechanics, Aeroelasticity, and Control”, Cambridge University Press, Cambridge, U.K., 2023.

[6]. R. Vepa, “Flight Dynamics, Simulation, and Control of Aircraft: For Rigid and Flexible Aircraft”, CRC Press, 2nd Edition, Boca Raton, FL, USA, 2023.

[7]. S. Kumar, W. Zhong, J. Williamson, P. Kumar, T. Furness, S. Lou, W. Zeng, and J. Jiang, “Design, Fabrication, and Testing of Freeform Mirror-Based Head-Up Display System”, Optics & Laser Technology, vol. 186, Article 112653, 2025.

[8]. S. Susindar, T. Ferris, J. Valasek, E. Fojtik, M. Brown, K. Pugh, C. Marcario, and T. King, “Operational Human Factors Considerations for Head-Worn Display (HWD) Usage in Civil Aviation”, FAA Office of Aerospace Medicine, Report DOT/FAA/AM-25/07, Washington, DC, USA, 2025.