

Algorithms and techniques for solving vehicle routing problems (VRP): A systematic review of the literature

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Abstract— The vehicle routing problem (VRP) has become one of the most used techniques in the treatment of logistic problems. Its uses and applications focus on designing the optimal routes for solving complex routing problems. This article makes a systematic review of the VRP. Analyzes and synthesizes information from 20 scientific papers focused on the development and application of algorithms and heuristics to solve routing problems. The articles were classified into four approaches: 1) Cumulative Vehicle Routing problems (Cum-VRP), 2) Routing problems, 3) Population-based optimization algorithms and 4) Machine Learning methods focused on route generation and optimization. The research was carried out to update the state of the current literature and identify existing gaps in the field of VRP. One of the main findings among the products of the first approach was that all of them defined an objective function which consisted minimizing costs, distances, travel and waste. After reviewing and analyzing the selected products, it was concluded that to solve logistic problems based on vehicle routing, it is better to use population-based approaches, such as: genetic algorithms (GA), particle swarm optimization (PSO), Memetic Algorithms (MA), brain storm optimization (BSO) and ant colony optimization (ACO), since they achieve better results in their development and application. In addition, it was also found that the mathematical models and exact approaches developed for this type of problem are limited as instances grow.

Keywords—VRP; Optimization; Routes; Review; Heuristics.

I. INTRODUCTION

There are currently many studies in the literature that focus on various aspects of logistics processes, such as network design, economic and environmental performance, batch size, return forecast and vehicle routing [1]. Among these activities, the vehicle routing problem (VRP) is the most complete. This is the process that includes both logistics operations and transport systems [2]. [3] defined VRP as the process of distributing or collecting certain customer groups or products with a vehicle or groups of vehicles by

creating optimal routes. Researchers have been working on this process since 1959 [4] and over the last six decades many variants of VRP have been developed, modelled and solved [5].

II. SYSTEMATIC LITERATURE REVIEW

Throughout history, the scientific community has contributed to the solution of routing problems with a wide variety of articles, both research and review. This section presents information from 20 scientific products on the VRP and highlights important factors such as the relationship between them, their approaches, differences and similarities.

20 scientific products were selected and reviewed for the development of this section. They were classified into four approaches for their analysis: Cum-VRP, Routing, Optimization Algorithms and Machine Learning. The classification is shown in Table I. See Table I.

TABLE I. CLASSIFICATION OF REVISED ARTICLES

Research approaches	Number of articles reviewed
Cum-VRP	5
Routing	4
Optimization algorithms	4
Machine Learning	7

Source: Authors

Table I shows the classification of the 20 articles reviewed in this section. 5 articles focusing on cumulative routing troubleshooting, 4 articles focusing on routing troubleshooting, 4 articles on the most used optimization algorithms for solving routing problems and 7 articles that used Machine Learning techniques to solve this problem. Each approach is described and analyzed in a particular way in sections A, B, C and D respectively.

A. Cumulative Vehicle Routing Problems (Cum-VRP)

Cumulative Vehicle Routing Problems (Cum-VRP) are an extension of the classic VRP [4]. Cum-VRP aims to find a set of delivery routes that optimize a given objective function. This type of routing problem has attracted the interest of researchers and professionals due to its wide field of possible applications. Several research papers have been

generated from this with the aim of continuing and expanding its study.

An information table was developed which includes the analysis of 5 scientific products reviewed in this section, which talk about solving problems of cumulative vehicle routing. Table II mentions the author or authors, the title and their respective findings. See Table II.

TABLE II. ARTICLES FOCUSED ON CUM-VRP

Author	Title	Findings
[5]	<i>Cumulative vehicle routing problem</i>	They proposed the first Cum-VRP as a way to incorporate the flow of goods along routes.
[6], [7]	<i>Constructive algorithms for the cumulative vehicle routing problem with limited duration</i>	They considered the case of total load transported per vehicle and incorporated coefficients for load and distance into the target function.
[8]	<i>A metaheuristic approach for the cumulative capacitated arc routing problem</i>	They considered a Cum-VRP with an objective function that incorporates the weight of the vehicle multiplied by the distance traveled, the accumulated load of the vehicle and the cost related to demand.
[9]	<i>A memetic algorithm for the cumulative capacitated vehicle routing problem including priority indexes</i>	They expanded the problem by incorporating priority indices to cover business contexts such as the delivery of perishable goods.
[10]	<i>Cumulative VRP with Time Windows: A Trade-Off Analysis</i>	They investigated the Cum-VRP with hard and soft time windows to analyze the balance between environmental costs and customer dissatisfaction.

Source: Authors

According to Table II it was observed that the 5 products established an objective function to solve their respective routing problems. In addition, an important relationship was observed between the contributions of these authors, which in turn are presented chronologically and can be explained as

follows: First, in 2008, [5] focused on incorporating and servicing the flow of goods throughout the route designed, then in 2016 and 2017, [6] and [7] added to the above equation the load and distance coefficients in order to be able to analyze the capacity of the delivery trucks and quantify their fuel consumption. For the following year, in 2018, [8] they paid special attention to vehicle weight and multiplied it by the distance travelled to estimate costs corresponding to demand for the products. For their part, [9] in 2020 they met the priority indices, considering the preferential distribution of perishable products and finally, in that same year, [10] included two factors of great importance for the Cum-VRP approach, a quantitative factor and a qualitative one. They analyzed the environmental costs and customer dissatisfaction in designing optimal routes.

The focus on the Cum-VRP solution has expanded considerably over the years. The authors mentioned together with many others have contributed to this cause with their important scientific contributions.

B. Routing Problems

As many authors have contributed to the study of the Cum-VRP problems, there is also research that focuses on the application of new knowledge and tools to solve vehicle routing problems with their different variants.

A table showing the 4 scientific products that were classified to analyze this approach was made. The table shows the names of the authors, the year of publication, the titles of the articles and the findings of each. See Table III.

TABLE III. ARTICLES FOCUSING ON ROUTING PROBLEM SOLVING

Author	Title	Findings
[5]	<i>Cumulative Vehicle routing problem</i>	They implemented a model that focuses on the case of energy collection by means of car traffic and is tested in realistic instances using real data from the road map of Turkey.
[11]	<i>A multi-start algorithm with intelligent neighborhood selection for solving multi-objective humanitarian vehicle routing problems</i>	They analyzed the actual case of a flood in Villahermosa, Mexico in 2007 that affected much of its territory. To solve this problem, they developed a model capable of defining evacuation routes for the victims while minimizing waiting times.
[12]	<i>Iterated clustering optimization of the split-</i>	They applied an ant colony optimization system (ACO) to solve the routing problem of a customized bus

	<i>delivery vehicle routing problem considering passenger walking distance</i>	service, in order to optimize the distance traveled by passengers and minimize operational costs for the company.
[13]	<i>Nature-Inspired Optimal Route Network Design for Shared Autonomous Vehicles</i>	They addressed the design of an optimal route network for autonomous vehicles, applied an ant colony optimization algorithm to the problem to build a set of routes to meet user requests under operational restrictions. Their results showed that the algorithm can produce solutions in relatively short computational times, and that it also exploits the possibility of sharing trips to reduce operational costs.

Source: Authors

The 4 products shown in Table III are related to each other because they solve a routing problem or variant thereof. They apply knowledge in real cases with real data and focus on optimizing an objective function as specified by the Cum-VRP.

For example, to test the Cum-VRP formulation, [5] tested the technique in real cases using data from the Turkish road map and analyzing the effect of energy consumption compared to the number of vehicles used. Trials were conducted in 24 and 31 cities, with the number of trucks ranging from 4 to 9. The results indicated that the increase in the number of vehicles decreases the total energy used.

According to Table III the four products reviewed, in the first instance, designed a mathematical method for modelling a routing system with the purpose of optimizing a given objective function. Later, they applied it to a pre-established case study where they used real data in a controlled environment and were able to alter the conditions of the model to analyze the behavior of the data. Once the results were obtained, they wrote down their findings and published their contributions. The articles reviewed in this section, taken together, show real applications that solved similar vehicle routing problems with a high degree of accuracy.

C. Population-based optimization algorithms

Four major articles were chosen with this approach. Three research and one review. In the latter was presented the study of 42 publications that talk about the most used methods to solve both cumulative vehicle routing problems (Cum-VRP) and cumulative vehicle routing problems (CCVRP). The analysis of the review article is discussed in detail in subsection C.1

with the aim of better understanding the points highlighted by the author. The data in the documents set out in this section are shown in Table IV. See Table IV.

TABLE IV. ARTICLES THAT USED OPTIMIZATION ALGORITHMS TO SOLVE ROUTING PROBLEMS

Author	Title	Findings
[14]	<i>Research on the vehicle routing problem with Interval demands</i>	They focused on a vehicle route generation problem with interval demands (VRPID) and solved it by combining a programming method based on non-linear intervals and a hybrid algorithm based on the artificial immune system, achieving better results than a Genetic Algorithm (GA).
[15]	<i>Vehicle routing with cumulative objectives: A state of the art and analysis</i>	Found that 55% of the reviewed authors implemented solution approaches based on genetic algorithms and the remaining 45% contributed with particle swarm optimization (PSO), ant colony optimization (ACO) algorithms and Brainstorm Optimization (BSO).
[16]	<i>An effective evolutionary algorithm for the cumulative capacitated vehicle routing problem</i>	They presented the first metaheuristic designed for the Cumulative Capable Vehicle Route Problem, considering specific properties such as capacity constraints and a homogeneous fleet of vehicles to improve speed and efficiency. The algorithm they obtained was considered as the best metaheuristic for the itinerant repairer problem.
[17]	<i>Heuristic solution approaches for the cumulative capacitated vehicle routing problem</i>	They compare a genetic algorithm (GA) and the taboo search (TS) in terms of CPU time required and target values obtained for the CCVRP solution. They also propose a solution based on the nearest neighborhood.

Source: Authors

Based on Table IV it was mentioned that [15] published a review article, in which they reviewed 42 publications related to the resolution of problems of cumulative vehicle routing (Cum-VRP) and problems of cumulative vehicles routing (CCVRP). In order to deepen the methods proposed by the authors mentioned in this paper, the article was taken as a reference. The most important points of discussion

were collected and described in section C.1. For their part, to test the CCVRP formulation, [16] provided some properties of the problem, namely: 1.- the traveler repairer problem does not provide a lower limit for CCVRP. 2.- optimal use of the CCVRP exactly min $\{|K|, n\}$ and 3.- a route gets a different cost when reversed.

C.1. Key Background in the literature review on cum-VRP

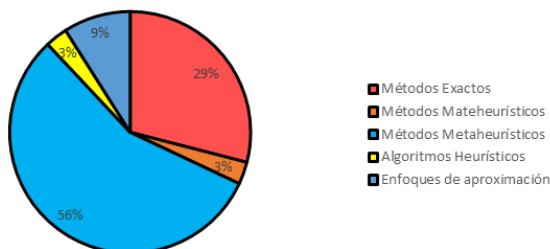
In [15] they published an article entitled "Vehicle routing with cumulative objectives: A state of the art and analysis" in the journal "Computers & industrial Engineering". This product was accepted for publication on February 28, 2022, and has been available online since March 5 of the same year.

After a thorough reading and analysis of the above-mentioned article, attention was focused on page 5 of the document, where the authors listed 42 publications and ranked them according to the methods used in each.

This table shows the exact and approximate methods that different authors used to solve Cum-VRP and CCVRP problems. Fig. 1 represents the distribution of works according to this classification. See Fig. 1.

Based on the results presented by [15], 17 of the 42 articles used exact methods, i.e. 15 MILP and 2 B&C&P. In most of these documents it is agreed that the MILP model is provided when a new variant of CCVRP and Cum-VRP is introduced. Among the 37 works which proposed approximation procedures, 2 heuristic algorithms, 33 metaheuristic methods and 2 matheuristic methods were developed. To better explain this, Fig. 1 shows the distribution by type of solution method. See Fig. 1.

Fig. 1. Distribution by type of troubleshooting method for routing problems



D. Machine Learning

Seven recent products focused on addressing routing problems using machine learning (ML) techniques were selected. Six papers and one master's thesis. In these products the authors developed techniques such as neural networks and optimization algorithms combined with machine learning models. The objective of these models was to

improve the efficiency of conventional heuristics. As well as trying to fill existing gaps in the current literature. The information in the documents set out in this section is shown in Table V. See Table V.

TABLE V. MACHINE LEARNING TECHNIQUES FOR ADDRESSING ROUTING PROBLEMS

Author	Methods
[18]	RBNN y K-means
[19]	LB y K-means
[20]	GA y GCN
[21]	GCN y AM
[22]	DNN y RL
[23]	MODRL-SIA
[24]	PMOCO Y DRL

Source: Authors

The thesis presented by [18] shows an interesting application of the development of radial-based neural networks to classify and optimize maintenance routes of public and school nodes with internet access, of the Digital Divide Reduction Program (DBRP). In this work the author used the k-means algorithm to train the hidden layer of the network and the Backpropagation algorithm to train the output layer. The author mentioned that the percentage of performance in her neural network with 3 centroids was 100% in all its stages, while in the training stage of the network with 6 centroids the performance dropped to 99.145299%. On the other hand, [19] proposed a constructive cluster-first, route-second heuristic method based on continuous approach (CA) for "one to many" vehicle routing for dispatching goods after an emergency. The authors investigated and compared two clustering methods: Local-based and K-means. After submitting a case study in Miami-Dade County, Florida, to dispatch fuels from the depot up to 72 service stations concluded that the Local-based clustering approach achieved lower total cost with higher movement cost. On the other hand, [20] proposed a joint approach based on genetic algorithm (GA) and graph convolutional network (GCN) to solve the problem of routed vehicles with multiple distribution centers. First, they used the heuristic method to modularize the complex environment and code each module according to the constraint conditions. Next, they adopted the convolutional network of graphs for embedding features for graph representation of the vehicle routing problem and used multiple decoders to increase the diversity of the solution space. Its experimental results showed that, compared to the GCN-based resolution method of a single decoder, the proposed method improves the resolution success rate to 100% in 15 generated instances. Similarly, [21] use the GAT-AM model combining Graph Attention Networks (GAT) and Multihead Attention Mechanism (AM) to solve the problem of vehicle routes with stochastic travel cost (VRP-STC). Their experiments show that the advantages of GAT-AM become greater as the complexity of the problem increases, with the optimal solution generally unattainable through traditional algorithms within an acceptable period. For

their part, [22] proposed a combination of deep neural network (DNN) with reinforcement learning (RL) to solve the dynamic vehicle routing problem with immediate acceptance or rejection of orders. They integrated realistic variables such as dynamic order arrivals, time windows and capacity constraints to improve their relevance for practical scenarios. Through their experimental results, they concluded that their model is promising for addressing VRPD in real-life environments, which emphasized the importance of careful and efficient modeling. In [23] they presented a complex algorithm for the optimization of swarm intelligence (MODRL-SIA), based on deep reinforcement learning, as a solution to the problem of location and routing with time windows (LRPTWs) in distribution for location logistics in the cold chain storage of fresh agricultural products. His algorithm was meticulously tuned to three key objectives. Minimize the overall cost of distribution, reduce carbon emissions and mitigate the depletion of fresh agricultural products. Their experimental findings indicated that the suggested algorithm outperforms other algorithms in each computational instance, confirming its effectiveness in meeting its objectives. In addition, [24] propose a multi-purpose vehicle route optimization algorithm based on preference setting. Incorporate the weight adjustment method in PMOCO that can adapt to different approximate Pareto fronts and find solutions with better quality. They treat weight adjustment as a sequential decision process and train it through deep reinforcement learning. According to their experimental method they showed that their algorithm yields a 6% improvement compared to PMOCO with 20 preferences. Subsequently, [25] proposed a mixed integer based on replacement arcs to treat the multiple-trip variant, i.e., mtCCVRP. Replacement arches were used to replace trips of a multiple trip with a single trip. The authors also proposed valid formulation inequalities that were useful in reducing execution time when the model was solved by a commercial solver. The model without valid inequalities solved 6 instances with up to 15 nodes, while the model with those inequalities led to 8 instances and reduced computational times. On the other hand, in a follow-up article for that same variant, two improved formulations, one based on flow and one based on set partitioning. In addition, the authors developed valid inequalities to improve model performance and an accurate procedure based on a resource-limited shorter path approach. The resulting problem was solved by an adaptation of the Bellman-Ford algorithm. Results indicated that the formulation resolved a minority of cases over 20 locations. On the contrary, the improved formulation could address instances with up to 40 locations, which shows that good initial solutions, dominance rules and lower limits improve the resolution procedure. Recently, [26] developed a mixed integer formulation for the multi-repository CCVRP, i.e., MDCCVRP, and proposed lower bound inequalities to solve instances of up to 10 nodes, while providing feasible solutions for instances of up to 100 customers. Considering the topology of

the problem (i.e., multiple deposits), the authors proposed a mathematical decomposition approach using the existing formulation. In addition, [27] proposed an entire linear mixed programming model for a variant of the MDCCVRP that incorporates mandatory visit times (MDCCVRmvt). The problem aims to minimize total delayed latency. To evaluate the performance of the proposed formulation, the authors generated 165 test instances with a size ranging from 10 to 50 nodes and between 2 and 4 deposits. The model could resolve instances with 10 and 20 nodes relatively easily, but in the case of instances with more than 40 nodes, the solver reached the time limit without finding the optimal solution in most cases.

III. CONCLUSIONS

This article shows a systematic literature review. Analyzes and synthesizes works that focus on the VRP solution and some of its variants such as the Cum-VRP and the CCVRP through the development of optimization algorithms, exact methods, approximation methods, among others. The development of such techniques allows routing problems to be solved in various fields of study.

The examples presented above represent some of the approaches that the authors have proposed to solve problems in the VRP field. They also represent the potential of the different methods used, since they obtained satisfactory results when developing them and showed them punctually in their work. The results that have been presented over the years give confidence to the scientific community and provide the basis for further research in the future.

According to the work reviewed in this article, population-based heuristics such as genetic algorithms (GA) and memetics (MA), particle swarm optimization (PSO), ant colony optimization (ACO) and brainstorm optimization (BSO) have been used to solve vehicle routing problems. However, the exact approaches to solving these problems are mainly MILP (Mixed Integer Linear Problems) formulations using CPLEX as the main engine. Another point to note is that, of the reviewed documents, nine also proposed approximate procedures due to the solvers' limitations in addressing medium- or large-scale scenarios.

From the above, it is concluded that, to solve logistic problems, based on vehicle routing, it is better to use approximation approaches. Because, in doing so, better results are obtained in their development and application and, moreover, these approaches allow them to address problems whose database is larger. It is also concluded that the mathematical models and exact approaches developed for this type of problem are limited as instances grow. In particular, the largest instance size solved to optimality is around 40 nodes. This justifies and explains the development of heuristics and metaheuristics to deal with large

instances. As for these procedures, there is a slight preference for the development of single-solution algorithms, since these approaches have proven their effectiveness in finding the BKS (best-known solution) for almost all legacy instances. In summary, the algorithms based on the neighborhood search are those that have reported the BKS on all instances. It is important to note that as the most recent metaheuristics are presented, the best-known solutions have been updated.

This systematic review provides the researcher with the ability to formulate objective arguments on a clear and specific basis, also helps you to scientifically support and sustain the development of scientific products with approaches in the field of vehicle routing or similar studies.

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