Tuning methods and controllers design for DC motor

Vasileios Provatas[1], **Spyros Provatas**[2] ,*
[1]Department of Production Engineer, Technical University of Crete, Greece,
Email: vasilisprovatas1998@gmail.com
[2]Department of Mechanical Engineer, University of Wets Attica, Greece,
Email: Spyrosprovatas@yahoo.com

Abstract: **This paper deals with the application of automatic controller compensation techniques to DC motors. DC motors are widely used in domestic, craft and industrial installations due to their simplicity and efficient operation. P, PI, PID controllers are classic control application elements, which are very widespread giving fast, reliable and easily adaptable solutions to a large group of devices that need control.**

Keywords: **DC Motors, Controllers, Ziegler-Nichols method, PID, Speed control.**

I. INTRODUCTION

DC motors are so named because they are powered by some source of DC voltage to provide the energy required to run them. From a construction point of view, they do not differ from direct current generators. Their main advantage is the ease of controlling their torque and speed over a wide range of values for its rated operating voltages as well as the loads that can be imposed on them.

In this paper, the mathematical modelling of the DC motor will be presented and then the development of controllers to control the angular velocity of the rotor will be carried out. Controller development will be [1] Ziegler-Nichols tuning method. Finally, graphs will be presented with the simulation of the controllers on the DC motor to observe the response of the controllers and draw conclusions.

II. MATHEMATICAL MODEL OF DC MOTOR

Every DC motor consists of two main parts:

[1] The stationary part is called the stator and
[2] The rotating part is called the rotor.

The stator has a circular shape and the rotor is contained inside. The stator is supplied with a continuous voltage in order to create a magnetic field so as to excite the rotor. This is how the rotation of the rotor is achieved due to the forces that will be exerted on it by the magnetic field. Figure 1 shows the inside of a DC motor.

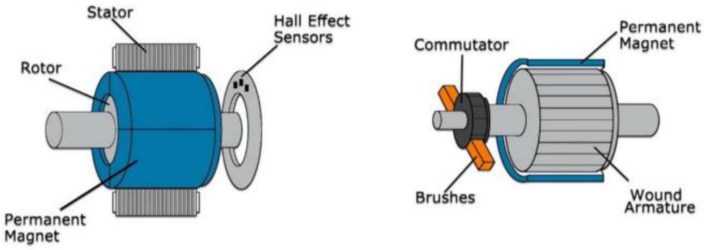

Fig. 1. DC motor

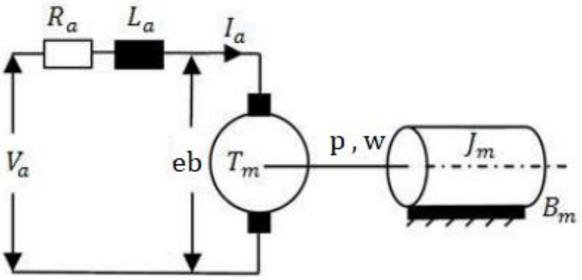

Fig. 2. Electrical diagram for DC motor

The motor torque:

$$\mathrm{Tm(t) = Kt\,Ia(t)} \qquad [1]$$

The relationship between the input voltage to the armature and the armature current:

$$\mathrm{Va(t) = Ra\,Ia(}t\mathrm{) + La\,\frac{dIa(t)}{dt} + eb(t)} \qquad [2]$$

The relationship between back electromotive voltage and motor speed:

$$\mathrm{eb(t) = Kb\,w(t)} \qquad [3]$$

The relation between motor torque and both load torque and disturbance torque:

$$\mathrm{Tm(t) = Jm\,\frac{dw(t)}{dt} + Bm\,w(t)} \qquad [4]$$

From [2],[3] :

$$Va(t) = Ra\,Ia(t) + La\frac{dIa(t)}{dt} + Kb\,w(t) \qquad [5]$$

From [1],[4]:

$$Kt\,Ia(t) = Jm\frac{dw(t)}{dt} + Bm\,w(t) \qquad [6]$$

The relations [5],[6] describe the model of a Dc motor in the field of time.

In relations [5],[6] a Laplace transformation is performed in order to find the transfer functions that describe the DC motor.

$$Va(s) = Ra\,Ia(s) + La\,Ia(s)\,s + Kb\,w(s) \qquad [7]$$

$$Kt\,Ia(s) = Jm\,w(s)\,s + Bm\,w(s) \qquad [8]$$

Solving relation [8] for *Ia(s)* and substituting *Ia(s)* into relation [7] yields the transfer function of the system,

$$Gp(s) = \frac{w(s)}{Va(s)} = \frac{Kt}{La\,Jm\,s^2 + (Ra\,Jm + La\,Bm)s + (Ra\,Bm + Kt\,Kb)} \qquad [9]$$

The relation [9] is between the rotor angular velocity and the armature voltage. The armature voltage (*Va(s)*) is the input of the system while the output is the angular velocity of the rotor (*w(s)*), as described in Figure 3.

Through the relationship,

$$\frac{1}{s}w(s) = p(s)$$

The relationship between armature voltage (*Va(s)*) and rotor position (*p(s)*) ,

$$\frac{p(s)}{Va(s)} = \frac{Kt}{La\,Jm\,s^3 + (Ra\,Jm + La\,Bm)s^2 + (Ra\,Bm + Kt\,Kb)s} \qquad [10]$$

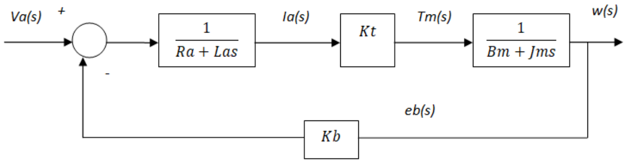

Fig. 3. Open-loop diagram of a DC motor

The state space model methodology is presented, using two state variables (x1 ,x2) ,

$$\begin{cases} x1 = w(t) \\ x2 = \dfrac{dw(t)}{dt} \end{cases}$$

$$\begin{cases} \dot{x1} = \dfrac{dw(t)}{dt} = x2 \\ \dot{x2} = \dfrac{d^2w(t)}{dt^2} = \dfrac{(-Ra\,Bm - Kt\,kb)}{La\,Jm}x1 + \dfrac{(-Ra\,Jm - La\,Bm)}{La\,Jm}x2 + \dfrac{Va(t)Kt}{La\,Jm} \end{cases}$$

Tables A, B, C, D from the state space model,

$$\begin{bmatrix}\dot{x1}\\ \dot{x2}\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{(-Ra\,Bm - Kt\,kb)}{La\,Jm} & \dfrac{(-Ra\,Jm - La\,Bm)}{La\,Jm}\end{bmatrix}\begin{bmatrix}x1\\ x2\end{bmatrix} + \begin{bmatrix}0\\ \dfrac{Kt}{La Jm}\end{bmatrix}[Va]$$

$$[y] = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x1\\ x2\end{bmatrix} + [0]\,[Va]$$

where,

Va= armature voltage (V)
Ra= armature resistance (Ω)
La= armature inductance (H)
Ia= armature current (A)
ed= Back electromotive force (V)
w= angular velocity of rotor (rad/s)
p= angular position of rotor (rad)
Jm=rotor inertia (kg m^2)
Bm=viscous friction coefficient (N m s/rad)
Tm=motor torque (N m)
Kt=torque constant (N m/A)
Kb= back electromotive force constant (Vs/rad)

TABLE I. System data

Ra	3.3 Ω
La	0.00464 H
Bm	1.8 10^{-6} N m s/rad
Jm	9.64 10^{-6} kg m^2
Kt	0.028 N m/A
Kb	0.028 Vs/rad

In this paper the angular velocity of the motor will be checked so the transfer function of the process,

$$Gp(s)=\frac{w(s)}{Va(s)}=\frac{0.028}{4.47\ 10^{-8}s^2+\ (3.122610^{-6})s+(7{,}899410^{-4})}$$

III. CONTROLLER DESING

From the open-loop system we will go to the closed-loop system, as depicted in figure 4. Some controllers will be designed in order to optimize the system of a DC motor.

Controllers will be developed using one methods:

- Ziegler-Nichols tuning method

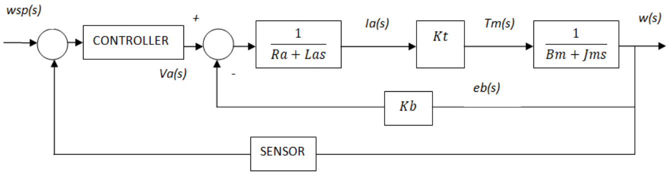

Fig. 4. Closed-loop diagram of a DC motor

The controller will receive data about the angular velocity of the rotor through some sensor, such as optical or magnetic pulse measurement, moving coil, etc., in order to change the armature voltage accordingly. The change in the armature voltage will also cause a change in the armature current which will help change the motor torque so that the angular velocity changes and converges towards the desired value.

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Summarizing the key features of the closed loop system:

- Reduces errors since the input is automatically adjusted
- Improves the stability of an unstable system
- It makes the system more resistant to external disturbances

IV. ZIEGLER-NICHOLS METHOD

Ziegler and Nichols described two methods for tuning the parameters of a PID controller. The two of them are the "Ziegler-Nichols' open loop method" and the "Ziegler-Nichols' closed loop method".

TABLE II. Tuning Ziegler-Nichols

Controllers	kc	ti	td
P	0.5 Ku	∞	0
PI	0.45 Ku	0,833 Pu	0
PID	0.6 Ku	0.5 Pu	0.125 Pu

Where, Ku is the critical gain and Pu the critical amplitude to be found through the geometrical place.

1. The analog parameter P,

$$P(t)=\mathrm{kc}\ \mathrm{E(t)}$$

2. The integral parameter I,

$$I(t)=\frac{\mathrm{kc}}{\mathrm{ti}}\int_0^{t}\mathrm{E(t)dt}$$

3. The differential parameter D,

$$D(t)=\mathrm{kc\ td}\,\frac{\mathrm{dE(t)}}{\mathrm{dt}}$$

The controllers where they will be developed with the aim of optimizing the system,

1. P CONTROLLER
2. PI CONTROLLER
3. PID CONTROLLER

PID controller output signal,

$$U(s)=kc\left(1+\frac{1}{ti\ s}+td\ s\right)E(s)$$

PI controller output signal,

$$U(s)=kc\left(1+\frac{1}{ti\ s}\right)E(s)$$

P controller output signal,

$$U(s)=kc\ \ E(s)$$

Fig. 5. Output signal and input signal of controller

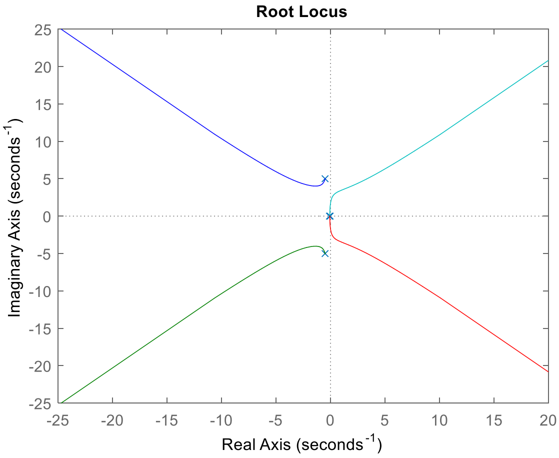

Fig. 6. Geometrical place

TABLE III. Tuning Ziegler-Nichols results

Controllers	kc	ti	td
P	11	∞	0
PI	9.9	3.0608	0
PID	13.2	1.8372	0.4593

V. RESPONSE CHARACTERISTICS

TABLE IV. Characteristics

Feature	Description
Overshoot	Maximum point – set point
Delay time	The time it takes for the response to arrive, for first time, 50% of the final of the price
Rise time	The time it takes to increase the response from 10% at 90% of its final price and is given by the relation
Peak time	Maximum point response from “0”
Maximum lift time	Time until the response reaches the maximum point
Settling time	The time it takes the response to be within the 2% or 5% range of the set point value
Dead time	zero error between the desired value and the system response

VI. CONTROLLER SIMULATION

With the help of Matlab, the controllers are developed in a DC motor with the aim of observing the variation of the response of the output value in relation to the set point. More specifically, all controllers will try to converge the angular velocity of the DC motor to the desired angular velocity defined by the user. The goal of the controller is to operate the motor at the desired speed with zero error.

P controller,

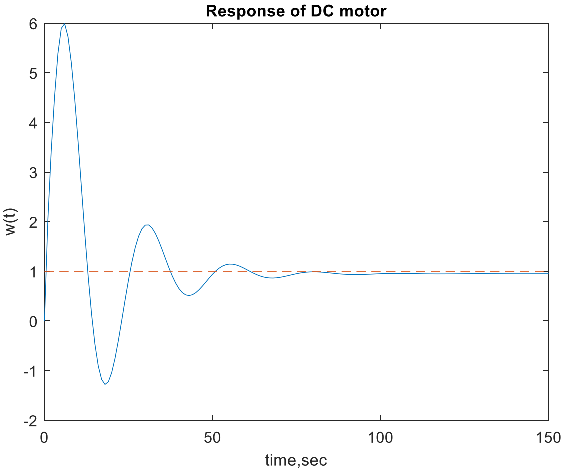

Fig. 7. Response of P controller

Figure 7, shows the response of a DC motor using a P controller. It is observed that the controller converges to the desired value without zeroing the error between the angular velocity of the motor and the desired value. The system response follows a decreasing oscillation and ends up in a quiescent state at 75 seconds.

TABLE V. Characteristics of P controller

Overshoot	4.9934
Delay time	0.25
Rise time	0.35
Peak	5.9934
Peak time	5.9
Settling time	-
Dead time	-

PI controller,

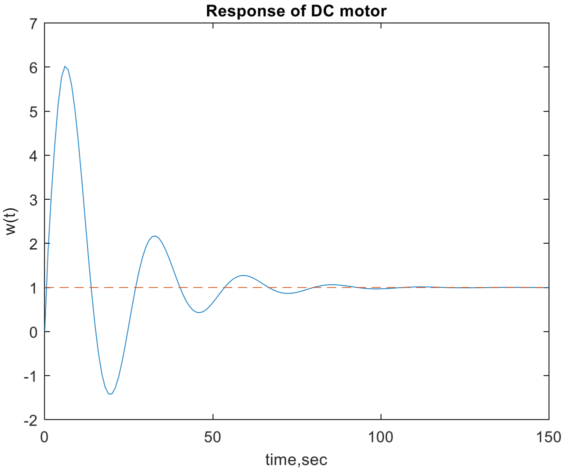

Fig. 8. Response of PI controller

Figure 8, shows the response of a DC motor using a PI controller. It is observed that the PI controller has a similar acquisition to the P controller. The difference is that the PI controller zeros out the error between the desired value and the angular velocity of the motor. The system response follows a decreasing oscillation and ends up in a quiescent state at 80 seconds

TABLE VI. Characteristics of PI controller

Overshoot	5.0232
Delay time	0.25
Rise time	0.35
Peak	6,0232
Peak time	6,1
Settling time	118,1
Dead time	133

PID controller,

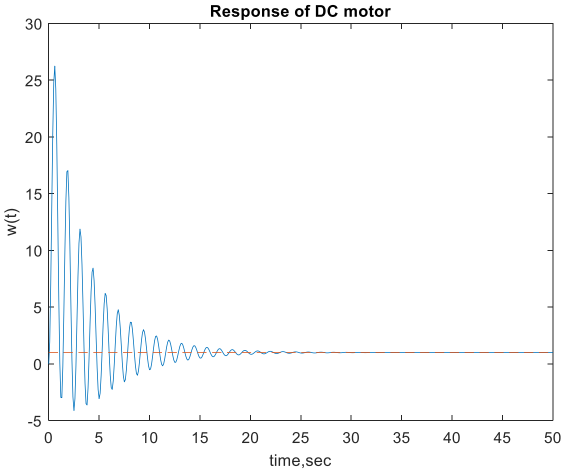

Fig. 9. Response of PID controller

Figure 9, shows the response of a DC motor using a PID controller. It is observed that the PID controller presents the best response in relation to the other two controllers, because it converges faster to the desired value of the angular velocity, in 25 to 30 seconds and presents zero error.

TABLE VII. Characteristics of PID controller

Overshoot	25
Delay time	0.025
Rise time	0.035
Peak	26
Peak time	0.6
Settling time	25,3
Dead time	35

VII. CONCLUSION

The designed PID has much faster response than response of the P and PI controllers. The PID is much better in terms of the rise time and the settling time than the others controllers. In essence, the PID controller responds faster to the reference signal we have set, that is, it drives the motor faster to its desired operating speed.

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