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Modeling of Blood Flow in an Aorta

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***Abstract*— in this paper, a mathematical model has been developed to study the velocity, flow and pressure gradient in an aorta. The aorta under consideration for the development of model is cylindrical with an axially symmetric dilation. The flow of blood ejected from the valve and flow to fill the entire aorta considered to be laminar. The results show that, in the dilation part of aorta, the gradient pressure and velocity are having irregular relation with the size of dilation. Also it's observed that after the peak velocity reached, the gradient pressure increases, the flow of blood reversed direction and the flux is stopped three times along the dilation part of aorta. Finally the flow of blood in the dilation part of aorta can be disturbed but in the normal part of aorta is undisturbed**

***Keywords*— bloodflow; wall shear stress; pressure gradient; viscosity**

I. INTRODUCTION

In the normal case of human circulatory system, the amount of pumped blood from the heart equals the amount of the entering blood to the heart. Existing the diseases such stenosis and aneurysm leads to obstruct the flow of blood from and to the heart, but the blood flow parameters such velocity, pressure, viscosity and wall shear stress will changes to save the same amount of pumped blood from and to the heart. An irregular relation between the wall shear stress and the size of aneurysm have been indicated (Musad 2016).

The total volume of blood is about 4.5–5.0 L, so all the blood is pumped throughout the body every minute or about 80 cm^3/s = 4.8 L/min. The flow rate in the arteries, arterioles, capillaries, venules, and veins are all the same according to Irving (2007).

The viscosity of blood is depends on velocity, temperature and the size of blood vessel. In small blood vessels and at high velocities, blood viscosity apparently decreases with decreasing vessel size and it begins to play a role in vessels smaller than 1 mm in diameter (Fahraeus -Lindqvist effect). In principle, the pressure drop over a blood vessel and the flow through it, together with vessel size, can be used to derive viscosity on the basis of Poiseuille's law Marc (2008)

Clearly, blood flow in the large arteries can be disturbed or turbulent. Disturbances have been observed in the ascending aorta of man, in the descending of dogs and rabbit, and as far as the thoracic artery horses, Nerem et al (1972) (1974).

II. DEVELOP THE MATHEMATICAL MODEL

The aorta under consideration for the development of model is cylindrical with an axially symmetric dilation. The flow of blood ejected from the valve of aorta is diffused and flow as study and laminar to fill the entire aorta.

The geometric of the assuming aorta and its coordinates system is given in figure (1) and equation (1).

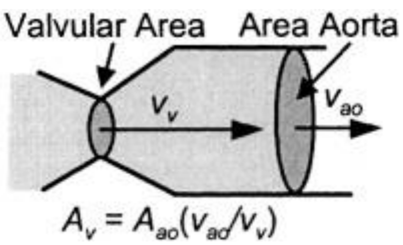

Figure 1.The boundary surface of dilation aorta taken from Marc (2008)

$$R(z) = r\,(1+\frac{R}{2r}COS(\pi\frac{z_0}{z}))), \quad -z_0 < z < z_0 \qquad (1)$$

Where R and r are the radii of aorta and valve respectively z_0 is the length of dilation part of aorta

There are two forces $2\pi rP$ and $2\pi\frac{\partial}{\partial r}(r\tau)$ force and surface force, τ is wall shear stress

Based on the boundary surface and the motion in cylindrical polar coordinates, the two forces can be balanced as

$$2\pi rP = 2\pi\frac{\partial}{\partial r}(r\tau)$$

$$\frac{\partial}{\partial r}(r\tau) = \mathrm{Pr} \qquad (2)$$

Integrate Equation (2) with respect to r, and divided both side by r we get

$$\tau = \frac{1}{2}\mathrm{Pr} + \frac{c}{r} \qquad (3)$$

At r=0 wall shear stress is finite then c =0, and we get

$$\tau = \frac{P}{2}r \qquad (4)$$

On such boundary surface the velocity is functions of two variables r and z, and then the velocity of blood across any section of aorta can be drive from equation (4) as

$$\frac{dv}{dr} = \frac{P}{2\mu} \int_{0}^{R(z)} r dr$$

$$v = \frac{p}{4\mu} R^2(z) \quad (5)$$

Equation (5) is the equation of the velocity where P is gradient pressure and μ is the viscosity of blood

By using the continuity equation of flow

$Q = v_1 A_1 = v_2 A_2$ (6) Q=0 at A=0, we get

$$Q = \frac{\pi p}{4\mu} R^4(z) \quad (7)$$

Equation (7) can be solved for p to get

$$P = \frac{4\mu Q}{\pi r^4 ((1 + \frac{R}{2r} COS(\pi \frac{z_0}{z})))^4} \quad (8)$$

Equation (8) is the equation of pressure gradient P, μ is the viscosity and Q is the flow rate of blood across an aorta.

III. THE RESULTS AND DISCATION

Equation (8) has been solved by using mathematical software program called Microsoft Math, where (r=0.75cm), (R=2r), (z_0=2cm), (Q=90cm^3/s), and (μ=0.04dynes.s/cm^2). The results in table (1) show that gradient pressure has irregular relation with the size of dilation, but in the normal part of aorta is decreased. Also from table (2) it's clear that the velocity is irregular and reached to zero in three difference location of dilation part of aorta, but in the normal part is not. Also from figure (2) and figure (3) we observed that the peak velocity is reached, the gradient pressure increases, the forward flow decreased due to the reversed direction. This leads to disturbance the flow of blood in the dilation part of aorta, but in the normal part is not.

Table 1: Data of gradient pressure on the dilation and normal part of Aorta

Dilation	Part	Normal	Part
Length	P.G mmgh	Length	P.G mmhg
0.44	0.011	2.48	9.89
0.88	0.0013	7.33	0.00145
1.11	0.001	12.18	0.00089
1.33	0.011	17.03	0.00078
1.55	0.541	21.87	0.00074
1.77	323.7	26.72	0.00072
-	-	31.57	0.00071
-	--	36.42	0.00070
-	-	41.27	0.00069
-	-	46.12	0.00069
-	-	50	0.00138

Table 1: Data of velocity for mean velocity equals 30cm/s

Length of dilation	v/v_0	Velocity
0.22	0	0
0.44	1.00	30.0
0.66	0.00	0.00
0.88	2.91	87.4
1.11	3.27	98.2
1.33	1.00	30.0
1.55	0.14	4.25
1.77	0.005	0.17
2	0	0

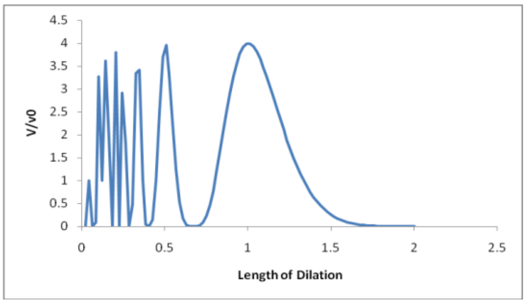

Figure 2: distribution v/v_0 along the dilation part of aorta

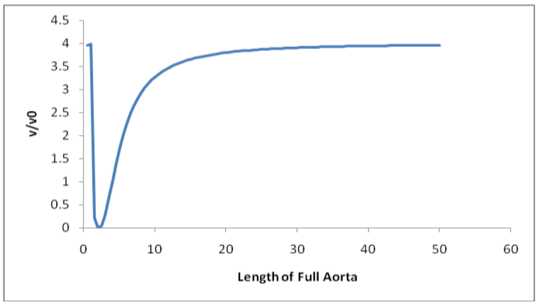

Figure 3: distribution v/v_0 along the dilation and normal part of aorta

IV. CONCLUSION

This mathematical study is given accurate description of blood flow though aorta and determined values of gradient pressure and velocity along with different length of dilation and normal parts of aorta.

The peak velocity reached, the flow of blood decreases due to increases the gradient pressure which increases the flow reversed direction. Then the flow of blood, in dilation part of aorta, is classified to be disturbed. These results have very good agreement with the experimental result by Nerem et al (1972) and 1974). But here we mentioned the reasons of disturbed, also by using this model we can predict where this disturbed appeared and finished.

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