

Research On The Spreading Model Of Stock Market Rumors On Homogeneous Network

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Abstract—*In the stock market, rumor spread has a great impact on the stock market volatility, and there are similarities between rumor spread and infectious diseases. On this basis, a rumor propagation model of ISRI stock market is established. By using the method of infectious diseases, this paper studies and discusses the stability of the basic regeneration number R_0 , disease-free equilibrium point and risk equilibrium point of stock market rumors on the traditional uniform network. It is found that when $R_0 < 1$, the disease-free equilibrium is locally asymptotically stable, and when $R_0 > 1$, the disease-free equilibrium is locally asymptotically stable. Finally, the correctness of the conclusion is verified by simulation. The results show that the rate of moving in and out of the stock market, the rate of losing immunity and the degree of network congestion have an impact on rumor spreading. The corresponding countermeasures are put forward.*

Keywords—*stock market rumor; homogeneous network; basic regeneration number; simulation.*

I. INTRODUCTION

With the rapid development of the Internet, the means for stock investors to obtain and exchange information has also changed a lot. Under the Internet mode, the speed and depth of information dissemination in the stock market have been greatly improved. On the other hand, because of the rapid development of the Internet, many harmful information is also mixed in the stock market information, which damages the stability of the stock market. Therefore, we must control the rumor in the investor network, so as to avoid the panic caused by the sharp fluctuation of the stock market. The spreading mechanism of stock market rumor is similar to that of social network rumor. In the early days, Daley and Kendall[1] pointed out the difference between the spread of infectious diseases and the spread of rumors, and put forward DK rumor propagation model. Thomson[2] modified DK model as MT model. Based on these early models, many researchers have studied the topological

properties of rumor propagation and related social networks[3-6].

On the basis of network topology, scientists began to seriously consider the role of rumors in human behavior and the spread of different mechanisms. Zhao[7] considered and analyzed the influence of forgetting rate on the dynamic mechanism of rumor propagation, and the results of numerical solution on livejournal showed that forgetting rate can affect rumor propagation. Xia[8] proposed a SEIR rumor propagation model considering hesitation mechanism. The conclusion shows that reducing rumor ambiguity can effectively improve the propagation threshold of SEIR model and minimize the influence of rumor. Wang[9] proposed a new Sir rumor propagation model by introducing the trust mechanism between the susceptible and the infected. They concluded that the introduction of trust mechanism not only greatly reduced the influence of rumors, but also delayed the end of rumors. In most previous studies, forgetting rate was considered a constant. In reality, however, the longer a rumor is held, the easier it is to forget it. Therefore, forgetting mechanism shows strong time dependence. Zhao[10] proposed a variable forgetting rate rumor propagation model based on exponential forgetting function, and discussed the influence of individual forgetting rate on rumor propagation when it changes with time. Wang[11] considered the sihr rumor propagation model of forgetting mechanism and memory mechanism, and concluded that forgetting and memory mechanism can affect the final rumor size. Wang[12] and others studied the theory of investor sentiment from the perspective of combining asset pricing and behavioral finance, indicating that the study of investor sentiment theory should not be limited to the study of the theory itself, but also should increase the research on the applicability and financial supervision. Komi[13] considers the influence of the group's education rate on the spread of rumors. The results show that the more educated people in the group, the smaller the final scale of rumors. Tihana[14] is the first attempt to combine theoretical overview with empirical tests for presence of herding effects in Croatia. Results indicate

that increase of market return leads to increase of market dispersion. However, changes of extreme values of market returns do not lead to herding behaviour, especially when the market is bearish. Kumar[15] build a cellular automaton model for the stock market to illustrate the complexity in the stock market by introducing variables for reflecting stability of the market. Rivera[16] presente articulo se analiza si la propagación de un rumor referido a las acciones en un mercado influye o no en el comportamient.

This paper mainly discusses the spread of stock market rumors on the investor network with the theory of infectious disease model. According to the fact that investors' exposure to stock market rumors is an emotional change, considering the situation of rumor spreading in stock investors' network, the corresponding dynamic equations of stock market rumor spreading are established on the uniform network. Matlab is used to simulate and analyze the influence factors of rumor and the influence of different parameters on rumor propagation.

II. THE ESTABLISHMENT OF THE RUMOR SPREADING MODEL IN ISRI STOCK MARKET

When the rumor spread in the stock market, it is similar to the rumor spread in the social network. In the investor network, due to the change of investor sentiment, the investors are divided into three types: the ignorant I, the disseminator S and the immune R. They respectively represent the investors who have not been exposed to the rumors of the stock market, the investors who have been exposed to the rumors and continue to spread the rumors of the stock market, and the investors who have lost interest in the rumors of the stock market and stop spreading the rumors. The state transition diagram is shown in Figure 1.

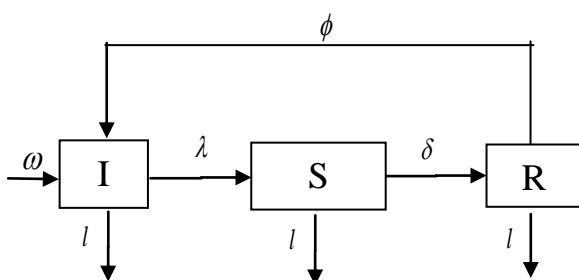


Fig. 1. Structure of rumor spread in the stock market

When the stock market rumor comes, the ignorant investor I will become the stock market rumor disseminator S with the probability of λ because he is more sensitive to the stock market information, thus affecting his mood fluctuation. Slowly, as the mood of the disseminator calms down, he becomes the

immune of stock market rumors with the probability of δ . Because the investor's sentiment is unpredictable, the immune person will lose immunity and become an ignorant investor with a probability of ϕ . When exposed to the rumors of the stock market, the above process will be repeated. Let's assume that ω is the new investor in the stock market and I is the group leaving the stock market.

According to the knowledge of complex network, different stock market investors are regarded as nodes, and the direct contact between stock market investors is regarded as the edge. First of all, we consider that the investor network is on a uniform network. We use $I(t), S(t), R(t)$ to represent the density of the ignorant, the disseminator and the immune respectively.

In addition, the following normalization conditions are met on the homogeneous network:

$$I(t) + S(t) + R(t) = 1 \quad (1)$$

According to the mean field theory, the ISRI dynamic model of the following homogeneous networks can be obtained:

$$\begin{aligned} \frac{dI(t)}{dt} &= \omega - \lambda \bar{k} S(t) I(t) - I I(t) + \phi R \\ \frac{dS(t)}{dt} &= \lambda \bar{k} S(t) I(t) - \delta S(t) - I S(t) \\ \frac{dR(t)}{dt} &= \delta S(t) - I R(t) - \phi R \end{aligned} \quad (2)$$

Here \bar{k} represents the average degree of the uniform network, where $\omega=1$. In the initial stage of rumor spreading in the stock market, there are only a few communicators in the network. We can get the following initial conditions: $S(0) = 0, I(0) = 1, R(0) = 0$.

III. MODEL ANALYSIS

A. Local stability and global analysis

For system (2), we can obtain the following equilibrium points:

$$E_0 = (1, 0, 0)$$

$$E_1 = \left(\frac{\delta + l}{\lambda \bar{k}}, \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{\lambda \bar{k}(\delta + \phi + l)}, \frac{\delta(\lambda \bar{k} - \delta - l)}{\lambda \bar{k}(\delta + \phi + l)} \right)$$

Theorem 1: $\text{sign } R_0 = \frac{\lambda \bar{k}}{\delta + l}$. When $R_0 < 1$, there is

always a disease-free equilibrium point $E_0 = (1, 0, 0)$ in system (2), and it is locally asymptotically stable. On

the contrary, e is unstable. When $R_0 < 1$, the system (1) has a unique endemic equilibrium point

$E_1 = \left(\frac{\delta + l}{\lambda \bar{k}}, \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{\lambda \bar{k}(\delta + \phi + l)}, \frac{\delta(\lambda \bar{k} - \delta - l)}{\lambda \bar{k}(\delta + \phi + l)} \right)$, And E_1 is globally asymptotically stable.

Prove: prove the above theorem by constructing Jacobian matrix of system (2).

$$J = \begin{bmatrix} -\lambda \bar{k} S(t) - l & -\lambda \bar{k} I(t) & \phi \\ \lambda \bar{k} S(t) & \lambda \bar{k} I(t) - \delta - l & 0 \\ 0 & \delta & -l - \phi \end{bmatrix}$$

Then E is locally asymptotically stable.

$$J(E_0) = \begin{bmatrix} -l & -\lambda \bar{k} & \phi \\ 0 & \lambda \bar{k} - \delta - l & 0 \\ 0 & \delta & -l - \phi \end{bmatrix}$$

$$\begin{aligned} & |J(E_0) - \mu E| \\ &= \begin{vmatrix} -l - \mu & -\lambda \bar{k} & \phi \\ 0 & \lambda \bar{k} - \delta - l - \mu & 0 \\ 0 & \delta & -l - \phi - \mu \end{vmatrix} \\ &= (-l - \mu)(\lambda \bar{k} - \delta - l - \mu)(-l - \phi - \mu) \end{aligned}$$

Obtain $\mu_1 = -l$, $\mu_2 = \lambda \bar{k} - \delta - l$, $\mu_3 = -l - \phi$.

if $R_0 < 1$, Namely $\frac{\lambda \bar{k}}{\delta + l} < 1$, $\mu_2 = \lambda \bar{k} - \delta - l < 0$,

Then there are $E_0 = (1, 0, 0)$.

Re proving that $E_0 = (1, 0, 0)$ is globally asymptotically stable.

Construct the Liapunov function $L(t) = aS(t)$, where $a > 0$.

Be

$$\begin{aligned} \frac{dL}{dt} &= a \frac{dS}{dt} = a(\lambda \bar{k} SI - \delta S - lS) \\ &< a(\lambda \bar{k} S - \delta S - lS) \\ &< aS(\lambda \bar{k} - \delta - l) \\ &< a(\lambda \bar{k} - \delta - l) \end{aligned}$$

Because $R_0 < 1$, that is $\lambda \bar{k} - \delta - l < 0$, that is

$$\frac{dL}{dt} < 0.$$

Downward syndrome at that time $R_0 > 1$,

$E_1 = \left(\frac{\delta + l}{\lambda \bar{k}}, \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{\lambda \bar{k}(\delta + \phi + l)}, \frac{\delta(\lambda \bar{k} - \delta - l)}{\lambda \bar{k}(\delta + \phi + l)} \right)$ it is

globally and gradually stable.

Evidence e is locally asymptotically stable:

$$J(E_1) = \begin{bmatrix} -\frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{(\delta + \phi + l)} - l & -\delta - l & \phi \\ \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{(\delta + \phi + l)} & 0 & 0 \\ 0 & \delta & -l - \phi \end{bmatrix}$$

$$\begin{aligned} & |J(E_1) - \mu E| \\ &= \begin{vmatrix} -\frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{(\delta + \phi + l)} - l - \mu & -\delta - l & \phi \\ \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{(\delta + \phi + l)} & 0 - \mu & 0 \\ 0 & \delta & -l - \phi - \mu \end{vmatrix} \\ &= \begin{vmatrix} -\frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{(\delta + \phi + l)} - l - \mu & \phi & -\delta - l \\ 0 & -l - \phi - \mu & \delta \\ \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{(\delta + \phi + l)} & 0 & 0 - \mu \end{vmatrix} \end{aligned}$$

We determine the eigenvalues of Jacobian matrix $J(E_1)$ by constructing polynomials:

$$\mu^3 + \sigma_2 \mu^2 + \sigma_1 \mu + \sigma_0 = 0$$

Where

$$\sigma_0 = (\lambda \bar{k} - \delta - l)(\phi + l)l$$

$$\sigma_1 = (\lambda \bar{k} - \delta - l)(\phi + l) + \frac{l(\lambda \bar{k} + \phi)(\phi + l)}{(\delta + \phi + l)}$$

$$\sigma_2 = 2l + \phi + \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{(\delta + \phi + l)}$$

Because $R_0 > 1$, that is $\lambda \bar{k} - \delta - l > 0$, we have

$$\sigma_0 = (\lambda \bar{k} - \delta - l)(\phi + l)l$$

$$\sigma_1 > (\lambda \bar{k} - \delta - l)(\phi + l) > 0$$

$$\sigma_2 > \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{(\delta + \phi + l)} > 0$$

$$\begin{aligned} \sigma_1 \sigma_2 - \sigma_0 &> (\lambda \bar{k} - \delta - l)(\phi + l)l \\ &\quad - (\lambda \bar{k} - \delta - l)(\phi + l)l \\ &= 0 \end{aligned}$$

So we say when $R_0 > 1$,

$$E_1 = \left(\frac{\delta + l}{\lambda \bar{k}}, \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{\lambda \bar{k}(\delta + \phi + l)}, \frac{\delta(\lambda \bar{k} - \delta - l)}{\lambda \bar{k}(\delta + \phi + l)} \right) \text{ is Locally asymptotically stable.}$$

So we conclude that

$$E_1 = \left(\frac{\delta + l}{\lambda \bar{k}}, \frac{(\lambda \bar{k} - \delta - l)(\phi + l)}{\lambda \bar{k}(\delta + \phi + l)}, \frac{\delta(\lambda \bar{k} - \delta - l)}{\lambda \bar{k}(\delta + \phi + l)} \right) \text{ is globally asymptotically stable.}$$

IV. NUMERICAL SIMULATION

In this section, the correctness of the above average field theory is verified by simulation, and the propagation characteristics of the proposed ISRI model on the uniform network are analyzed. In order to establish a better uniform network model, the number of nodes in the network is assumed to be $N=10^6$, and the average degree of the network is $\bar{k} = 10$.

Figure 2 shows the local asymptotics of the model around the equilibrium point E_1 with these parameter value sets. Among them $w = l = 0.0001$, $\lambda = 0.01$, $\delta = 0.01$, $\phi = 0.05$, $\bar{k} = 10$, it is known by calculation that $R_0 \approx 9.9 > 1$, rumors in the stock market will surely spread.

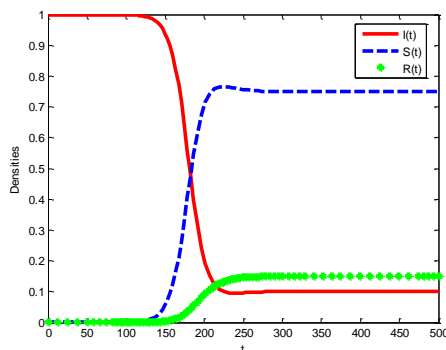


Fig. 2. The density of four groups over time at E_1

Figure 3 shows the change of the density of the nodes of propagation type $S(t)$ and immune type $R(t)$ with time t under different immigration and emigration rates l . It can be seen from Figure 3 (a) and (b): when

the value of the immigration rate l is larger, the density of the nodes of the propagation type $S(t)$ and immunity type $R(t)$ is smaller in the steady state, and therefore increasing the immigration rate l can effectively suppress the spread of rumors in the stock market. In the actual stock market, the rate of moving in and out represents the vitality of the market, which indicates that increasing the vitality of the market can weaken the spread of rumors in the stock market.

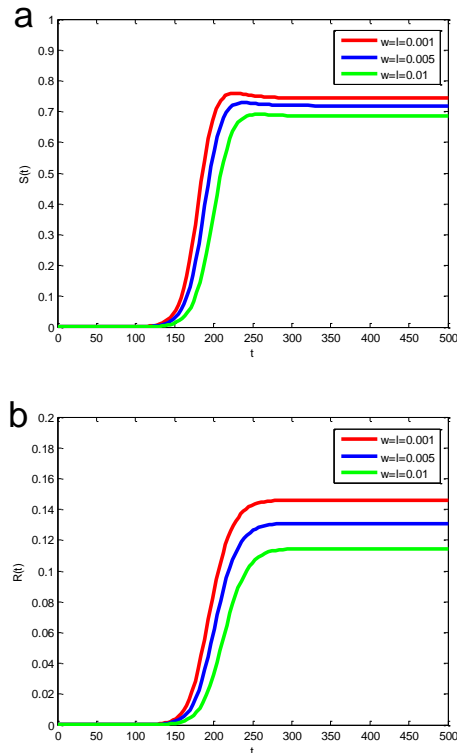


Fig. 3. The density of $S(t)$ and $R(t)$ changes with time t under different immigration rate l

Figure 4 shows the effect of transmission rate and immune rate on the basic regeneration number R_0 . From Figure 4 (a), we can see that the basic regeneration number R_0 is in direct proportion to λ . when the ignorant investors contact with the spreading investors, it is necessary to reduce the value of λ in order to control the spread of rumors. Therefore, it is necessary for the government or regulatory agencies to strengthen the management of the market and take corresponding measures to reduce the spread rate of rumors in the stock market. Figure 4 (b) shows that the basic regeneration number R_0 is inversely proportional to the immune rate of δ . increasing the immune rate can effectively inhibit the spread of rumors in the stock market.

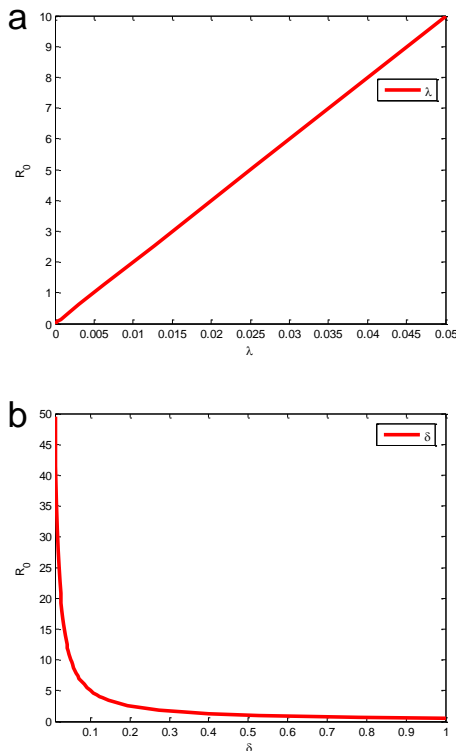


Fig. 4. The effect of transmission rate λ and immune rate δ on the basic regeneration number R_0

Figure 5 shows the change of $S(t)$ and $R(t)$ in different probability of losing immunity ϕ with time t . It can be seen from Figure 5 (a) and (b): with the increase of the probability of losing immunity ϕ , the steady-state density of nodes $S(t)$ increases, and the steady-state density of immune nodes $R(t)$ decreases. This shows that reducing the probability of losing immunity ϕ can reduce the density of steady-state communicators, which is helpful to control the spread of rumors in the stock market.

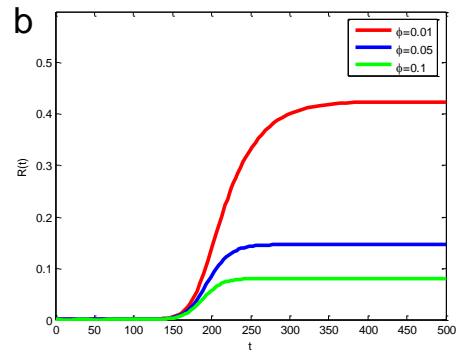
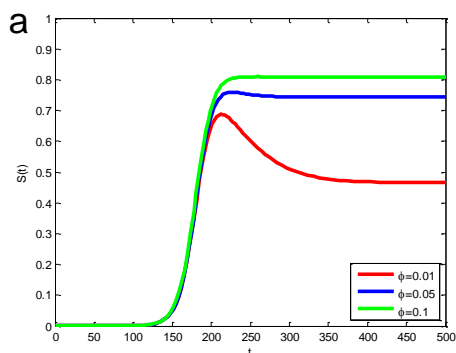


Fig. 5. The change of $S(t)$ and $R(t)$ in different probability of losing immunity ϕ with time t

Figure 6 shows the change of $S(t)$ and $R(t)$ density with time t for different k values, as shown in the figure below. It can be seen from Figure 6 (a) and (b): the higher the k value is, the higher the density peaks of $S(t)$ and $R(t)$ of the propagation type nodes are, and the faster the time to reach the density peak is, the higher the steady-state density of the propagation type nodes is. k value represents the average number of nodes contacted, and represents the investor's relationship in the investor network. That is to say, the more connected investors in the investor network are, the faster and the wider the rumor spread. Therefore, during the spread of stock market rumors, reducing the mutual contact between investors can effectively inhibit the spread of stock market rumors.

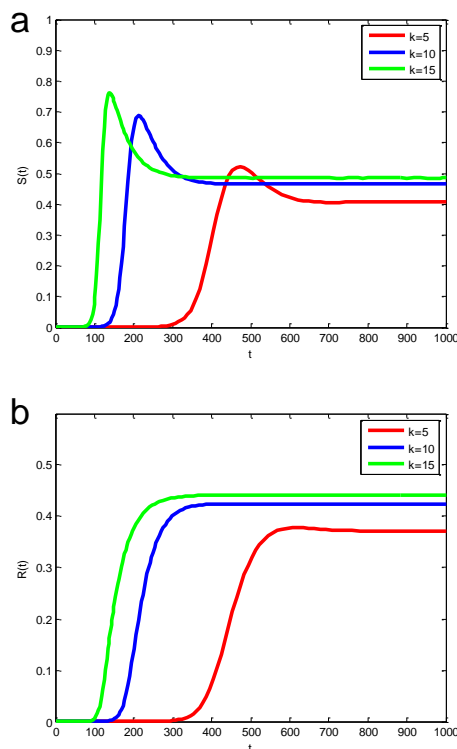


Fig. 6. The change of $S(t)$ and $R(t)$ density with time t for different k values

V. CONCLUSION

In this paper, the ISRS propagation model of stock market rumor with time delay on uniform network is established, and the basic regeneration number of stock market rumor propagation is obtained. At the same time, the non propagation equilibrium point and the propagation equilibrium point of the system are stable. It shows that if $R_0 < 1$, the rumor of stock market will not spread, and will disappear after a period of time; when $R_0 > 1$, there is a unique spread balance point, and the rumor of stock market will continue to spread. And through the simulation of MATLAB software, we get the following conclusions:

(1) When rumors spread in the stock market, market managers should take timely measures to stabilize investors' mood and increase the immune rate of the disseminators.

(2) It is of great significance to increase the liquidity of investors in the stock market as much as possible to stabilize the stock market.

(3) Compared with the investors with narrow connections, the investors with wide connections in the investor network have a higher probability of infection risk. Therefore, when the rumors spread in the stock market, we should try to reduce the contact between the investors.

(4) The smaller the immune losing rate in the immune stage, the stronger the inhibition of rumor in the stock market. Therefore, stock market managers should prevent immune investors from losing immunity, which is conducive to control the spread of rumors.

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