Demonstration Experiments on the Addition Of Unidirectional Harmonic Oscillations of the Same Frequency in The Study of Physics

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Abstract **– The article describes two devices that allow to trace the addition of two unidirectional harmonic oscillations of the same frequency. The first device allows to estimate from the sound intensity how the amplitude of the resulting oscillation changes depending on the difference in the starting phases of the summed oscillations. The second device uses the addition of two mechanical vibrations and allows you to trace the change in the amplitude of the resulting oscillation visually. The second device is intended for use with a document camera and a projector.**

Keywords— physics course; demonstration of experiment; addition of oscillations

I. INTRODUCTION

When studying harmonic oscillations in a physics course, the addition of unidirectional oscillations of the same frequency is often considered. Let's consider it in more detail [1].

If the first oscillation occurs as follows:

$$x_1 = A_1 \cos \omega t\ ,$$

where x_1 is the displacement of the oscillating material point from the equilibrium position,

A_1 – vibration amplitude,

ω - angular frequency,

t - time,

and the second oscillation occurs as follows:

$$x_2 = A_2 \cos (\omega t + \varphi),$$

where φ is the starting phase of the oscillation (in this case, it is also the difference between the starting phases of two oscillations) - then as a result of the addition of two such oscillations, the material point oscillates as follows:

$$x = x_1 + x_2 = A \cos (\omega t + \alpha),$$

where A is the amplitude of the resulting oscillation,

α - the starting phase of the resulting oscillation.

Here, we are more interested in amplitude A. The expression for it simplifies if $A_1 = A_2$.

In this case, A will be as follows:

$$A = A_1\sqrt{2(1 + cos\varphi)}$$

From this expression we see that if the oscillations occur in the same phase (in particular, if $\varphi = 0^0$), then:

$$A = 2A_1$$

With increasing φ, the amplitude A decreases, and if the oscillations occur in the opposite phase (in particular, if $\varphi = 180^0 = \pi$ rad.), then:

$$A = 0$$

Training material is undoubtedly much better absorbed if it is accompanied by demonstration experiments. You can, for example, demonstrate the addition of oscillations using two sound vibrations of the same frequency, if there is the possibility of changing the phase φ.

More obvious are mechanical vibrations, such as oscillations of a pendulum or arrows of an electromechanical indicator. In this case, students can directly see and evaluate the magnitude of the changing amplitude of the resulting oscillation with a change in phase φ.

This article describes two such devices.

II. DESIGN OF AUDIO DEVICE

This device allows to hear by ear the result of the addition of two oscillations, depending on the difference in their starting phases.

The electrical circuit of the device is shown in Fig.1 (see in the end of the article).

The harmonic generator is assembled according to the usual scheme on a chip XR2206 [2]. This chip is powered from series-connected Zener diodes VD1 and VD2 with a voltage of approximately 12 V. The whole device is powered by an 18 V DC adapter.

Using the trimming resistor R3, the voltage amplitude is set from the output of the generator. The maximum volume at the output of the device depends on this. Using the trimmer resistor R6, the frequency of this voltage is set within about 700….950 Hz. In original design it is equal to 800 Hz.

From the output of the generator, the voltage through the capacitor C5 is applied to switch SA2.1 (these are oscillations x_1). The same voltage is applied to the buffer with a voltage gain of one. It is assembled on ½ of LM358 chip, which consists of two operational amplifiers. The chip is powered by a voltage of ± 3.9 V from two Zener diodes VD2 and VD3. The second part of LM358 is a phase shifter assembled according to a known scheme (see, for example, [3]). Its voltage gain is also equal to one. It receives voltage from the output of the buffer. With the given value of potentiometer R11, the phase of the voltage at the output of the phase shifter can be changed within $30^0 \ldots 180^0$. This voltage (oscillations x_2) is applied to switch SA2.2. The starting phase of this voltage is also the difference between the starting phases of the two oscillations x_1 and x_2.

The choice of the starting phase is not from 0^0, but from 30^0, due to the desire to simplify the design. The calculation shows that with these two values of the starting phases (0^0 and 30^0) the difference in the volume of the perceived sound does not exceed 5...8%.

From the output of switch SA2, voltage is supplied to volume regulator R9. In the position 1 of the switch, an alternating voltage is applied to it, corresponding to the oscillations of x_1, in position 2 - corresponding to x_2, and in position 3 - $x_1 + x_2$.

The voltage from the volume regulator goes to the amplifier, assembled on TDA1519C chip. The amplifier provides additional voltage amplification of 40 dB and output power of several watts. In this design, with the help of resistor R3, the maximum power was chosen 1 W. The selected maximum power corresponded to the power rating of the applied loudspeaker with a resistance of 8 ohms. With the values shown in the circuit, the maximum power at which there was still no visible distortion of the sinusoidal voltage on the oscilloscope screen was equal to 1.5 W. With a decrease in the value of resistor R16, it can be significantly increased.

Potentiometers R9 and R11 are applied with a logarithmic characteristic (A type). The power of the used resistor R15 is 1 W , and of the resistor R16 - 5W. Using the trimming resistor R14, the equality of the amplitudes of the voltages corresponding to the vibrations x_1 and x_2 is established.

The appearance of the device is shown in Fig.2.

Fig.2. Appearance of the audio device

After turning on the device, you need to set the switch SA2 into position x_1 and select the desired sound volume using potentiometer R9. The volume regulator should not be touched anymore. Further, changing the position of the phase regulator, we show that both the sound frequency and its volume level stay unchanged. Turn the switch to position 2 and demonstrate that the sound has the same intensity and frequency as in position 1. We demonstrate that in this case, the magnitude of the starting phase does not affect the amplitude of the sound and its frequency. Next, put the phase regulator in position 30^0 and turn the switch SA2 into position 3. We attract the attention of listeners to the increase in the intensity of sound with constant frequency. Increasing the starting phase of the second oscillation (or, which is the same - increasing the difference between the starting phases of the summed oscillations), we fix a decrease in the perceived sound intensity. In particular, at a phase equal to 120^0, the intensity of the resulting oscillation does not differ from the intensity of each of the two oscillations (x_1 or x_2). This can be heard by moving the switch SA2 alternately in each of the three positions. Further increasing the phase difference in the third position of the switch, we hear that the sound volume continues to decrease. When the starting phase difference is 180^0, the sound disappears, i.e. the oscillations disappear.

The following should be noted here. If during a single oscillation (x_1 or x_2) an alternating voltage U is supplied to the loudspeaker, then the power is equal to U^2/R, where R is the resistance of the loudspeaker. When the starting phase difference of the summed oscillations is zero, the power on the loudspeaker increases by a factor of four when the oscillations are added. This is because the voltage acting now on the loudspeaker is 2U. But one should not expect that for a listener the intensity of the perceived sound will also increase four times. In accordance with the Weber-Fechner empirical law [4], it will increase approximately only by a factor of 1.5. That is why the second device, in which the addition of mechanical vibrations is used, is perhaps more informative.

III. DESIGN OF THE DEVICE WITH THE ADDITION OF MECHANICAL OSCILLATIONS

In this device, the arrow of a microammeter oscillates, with a zero at the middle of the scale. So that listeners could record the change in the amplitude of the oscillations, their frequency is chosen low. It is approximately 0.6 Hz. The device itself has dimensions 18 x 18 x 8 cm and is intended, like the devices described in the works [5, 6] to be jointly used with a document camera and projector.

The electrical circuit of the device is shown in Fig.3 (see in the end of the article). As we can see from this circuit, both devices have a lot in common.

The low-frequency oscillation generator is assembled according to the same scheme on XR2206 chip. The entire device is also powered by a DC adapter with a voltage of 18 V. The current consumed in this case does not exceed 50 mA. The power is turned on by a dual switch SA1.

The amplitude of the low-frequency voltage coming from the generator can be changed using the trimming resistor R3. The amplitude of the oscillations of the arrow of the PA1 microammeter is proportionally dependent on this amplitude. The voltage through the capacitor C2 is supplied to further elements of the circuit. Through the capacitor C5 it is supplied to the PA1. This voltage causes its arrow to oscillate in accordance with the expression:

$$x_1 = A_1 \cos \omega t ,$$

where x_1 is the shift of the arrow from the equilibrium position (in scale divisions).

Here, the starting phase of this oscillation is conventionally taken as zero. The same voltage after capacitor C2 is applied to the phase shifter assembled on ½ of the operational amplifier TS922. As in the first device, the phase shifter has a voltage gain of one. Using a potentiometer R8 (type A), the phase of the output voltage can be changed from almost zero to 180 degrees. The operational amplifier is powered by a bipolar voltage ± 3.9 V from Zener diodes VD3 and VD4. The voltage coming from the output of the phase shifter makes the arrow of the PA1 indicator to oscillate in accordance with the expression:

$$x_2 = A_1 \cos(\omega t + \varphi)$$

Thus, we have two harmonic oscillations with the same frequency ω and the same amplitude A_1. The difference in the starting phases between these oscillations can be changed in the range 0^0 ...180^0 (0 ….π rad.)

The trimming resistor R15 is used to precisely adjust the balance of the amplitudes of these two voltages.

As in the first device, the two mentioned voltages are supplied to the SA2 switch. In position 1, only the first voltage is applied to the microammeter PA1, causing the arrow to oscillate according to the law x_1. In position 2 - only the voltage leading to oscillations of the arrow according to the law x_2. In position 3, both voltages apply simultaneously and the arrow oscillates, respectively, according to the law $x_1 + x_2$.

A type 85C1 microammeter with a scale of 0 ....100μA and resistance of 1.33 kΩ was used as a indicator PA1. To move the arrow to the middle of the scale (to shift zero to the middle), a constant voltage of 3.9 V is used from the Zener diode VD3. This voltage is supplied through the trimming resistor R16 and the resistor R13 to the microammeter. Here we note that in the presence of a microammeter with a two-sided scale (with zero in the middle) it is easier to use it. Then the circuit with resistors R16 and R13 can be completely excluded.

In the process of adjusment using the trimming resistor R3, the amplitude of the oscillations x_1 (and, accordingly, x_2) is set to about ¼ of the microammeter scale. It is possible to precisely equalize the oscillation amplitudes of x_1 and x_2 using the trimming resistor R15. You may place the microammeter arrow into the center of the scale with the trimming resistor R16.

Before turning on the device, it is necessary to set the SA2 switch to the $x_1 + x_2$ position, and the phase regulator - to the 180^0 position. Next, you need to wait until the arrow calms down in the middle of the scale. This takes approximately 30 seconds. Now you can show the addition of oscillations.

We set the switch SA2 in the position x_1 and demonstrate the oscillations of the arrow with an amplitude in ¼ of the scale. By changing the position of the phase regulator in the range 0^0....180^0 we show that the oscillations do not depend on this.

Next, set the switch SA2 to the x_2 position. By changing the position of the phase regulator also within 0^0 ....180^0 (since the frequency of the arrow is small, we change the starting phase rather slowly!), we demonstrate that nothing changes. That is, neither the amplitude nor the frequency depend on this.

Now we transfer the switch to the $x_1 + x_2$ position. We show that the amplitude of oscillations at $\varphi = 0^0$ has doubled and amounts to ½ of the scale. That is, in this case, both oscillations occur in the same phase. Gradually increasing the phase difference, we see that the amplitude of the resulting oscillations decreases. At $\varphi = 120^0$, the amplitude again becomes approximately ¼ of the scale.

At $\varphi = 180^0$ amplitude becomes equal to zero, that is, the oscillations cease. Both oscillations are now in the opposite phase.

The appearance of the device is shown in Fig.4.

Fig.4. Appearance of the device using mechanical oscillations.

IV. CONCLUSION

The devices described above are schematically simple designs, which makes them easy to repeat. The first audio device allows you to evaluate by ear the change in the amplitude of the resulting oscillation obtained by adding two unidirectional harmonic oscillations of the same frequency when changing the difference in the starting phases of these oscillations.

The second version of the device allows you to visually trace the addition of two such mechanical vibrations on a large screen and quantify the change in the amplitude of the resulting oscillation when the difference in the starting phases of these oscillations changes. To do this, the device must be used jointly with a document camera and projector.

In our opinion, the use of these devices in physics lectures allows students to better understand the phenomenon of addition of unidirectional harmonic oscillations of the same frequency.

REFERENCES

[1] T.I.Trofimova „Physik“ Vieweg, Braunschweig/ Wiesbaden, 1997, pp. 194 – 195

[2] alldatasheet.com/datasheet-pdf/pdf/80496/EXAR /XR2206.html

[3] B.I.Goroshkov „Radioelektronnõje ustroistva“ Russia, Moskow, 1985; p.345

[4] en.wikipedia.org/wiki/Talk:Weber-Fechner_law# Sound_intensity

[5] Aleksei Gavrilov „Demonstration of the Lorenz Force Using a Spark Gas Discharge“ Journal of Multidisciplinary Engineering Science Studies, 2019, Vol.5, Issue 3, pp. 2545 – 2547

[6] Aleksei Gavrilov „The Study of Electrical Resonance in a Physics Course“ Journal of Multidisciplinary Engineering Science Studies, 2019, Vol.5, Issue 10, pp.2833 – 2836.

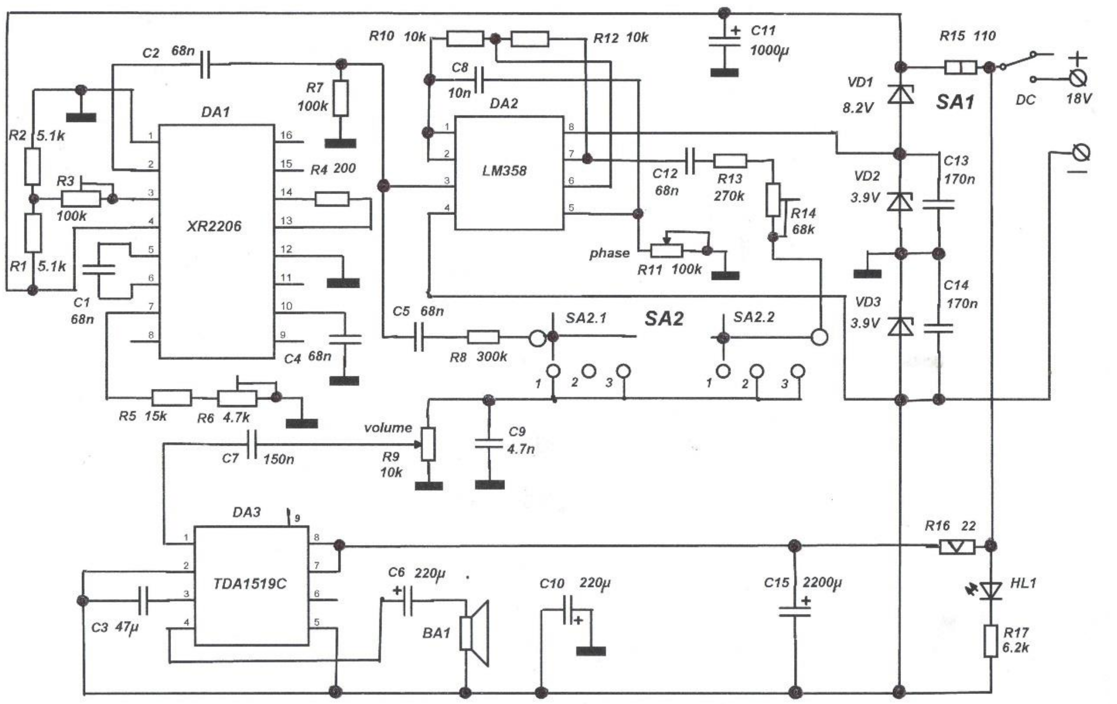

Fig.1. Electrical circuit of the audio device

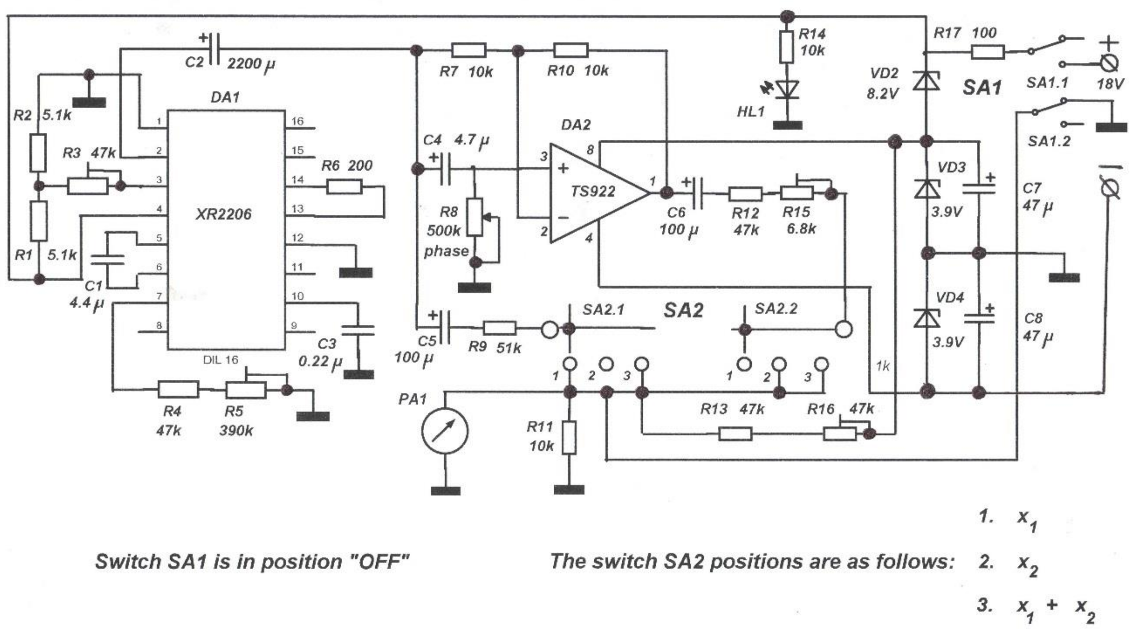

Fig.3. Electrical circuit of the device using mechanical oscillations.