Combined Effects of Hall Current, Rotation and Inclined Magnetic Field on a Free Convection Fluid Flow over an Exponentially Accelerated Vertical Plate with Heat and Mass Transfer

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***Abstract*—Many natural phenomena and technological applications undergo Magneto hydrodynamics (MHD). Applications such as the drawing of continuous strips of polymers through a die are carried out through a stagnant cooling fluid. The quality of the strips is found to depend on the rates of heat and mass transfer on the stretching surface. To assure quality, it is vital to understand how heat and mass transfer are affected by Hall current, rotation and inclined magnetic field for a free convection fluid flow. Due to wide applications of MHD in science and technology, many scholars have studied a wide variety of flow situations including those that combine Hall current, rotating systems and inclined magnetic field. However, no such studies have incorporated the effects on heat and mass transfer. In this study, we investigate the combined effects of Hall current, rotation and inclined magnetic field on a free convection fluid flow with heat and mass transfer for a fluid flowing over an exponentially accelerated vertical plate. A strong, steady and inclined magnetic field is applied into the fluid region. The coupled non-linear partial differential equations governing the flow are first expressed in dimensionless form then solved using the finite difference method. The skin friction and the rates of heat and mass transfer at each of the boundaries are computed using the least squares approximation method. Numerical values are simulated from the model equations using the MATLAB program. The profiles for velocity, temperature and concentration at various distances from the plate are demonstrated graphically for various parameters values. This study shows that increase in the angle of application of the magnetic field decreases the primary velocity while it increases the secondary velocity. The secondary velocity decreases when the Hall parameter is increased. It however increases when the rotation parameter is increased.**

Keywords—Hall current; Rotation effects; Inclined magnetic field; Exponentially accelerated Plate.

I. Introduction

Magnetohydrodynamics (MHD) is the study of the dynamics of electrically conducting fluids as they flow in a magnetic field. The MHD principle is employed by engineers in designing such devises as flow meters, MHD generators and pumps [5].

Situma *et al*. [6] researched on Hall current and rotation effects on MHD free convection flow past a vertical infinite plate under a variable transverse magnetic field. The study showed that temperature profiles decreased gradually near the plate but remained constant away from the plate. The work by Situma *et al*. [6] was extended by Thamizhsudar *et al*. [8] who considered Hall current effects on MHD flow of an exponentially accelerated plate relative to a rotating fluid with uniform temperature and mass diffusion. The results of the study showed that temperature of the plate decreased with increase in values of the Prandtl number while the concentration near the plate increased with decrease in values of Schmidt number.

An early study of the effects of suction/ injection and transverse magnetic field on the flow of a conducting fluid past an exponentially accelerated vertical plate was carried out by Kumar and Singh [3]. The magnetic field lines were taken to be fixed relative to the plate. The method of solution employed to solve the model partial differential equation was the Laplace transform technique. Deka [2] studied the effects of Hall current on hydromagnetic flow of a conducting fluid past an accelerated horizontal plate. The applied magnetic field was perpendicular to the flow. The method of solution applied was Laplace transform technique. The research did not incorporate heat and mass transfer due to orientation of the plates. Both the skin friction components along and perpendicular to the plate were found to increase as rotation parameter was increased.

Ahmed *et al.* [1] investigated MHD transient flow with Hall current past an accelerated horizontal porous plate in a rotating system under transverse magnetic field. Laplace transform technique was used to solve the model equations. The skin friction along the x-axis was found to decrease with increasing values of Hall current parameter. Sundarnath and Muthucumarswamy [7] however carried out studies on effects of Hall current on hydromagnetic flow past an accelerated plate with heat transfer. The effects of inclining the applied magnetic field were not considered in the study. When Hartmann number was increased, the axial and transverse velocity components increased in a direction opposite that obtained when the Hall current parameter and rotation parameter were increased.

Raju *et al.* [4] studied MHD flow over an exponentially accelerated infinite vertical plate. The fluid temperature was decreased due to heat absorption. This study too did not incorporate effects of inclined magnetic field. The study employed the use of Laplace transform technique and finite element method of solution. From the available literature, no research combining effects of Hall current, rotation and inclined magnetic field on a free convection fluid flow over an exponentially accelerated vertical plate with heat and mass transfer has been done. This is the motivation of this study.

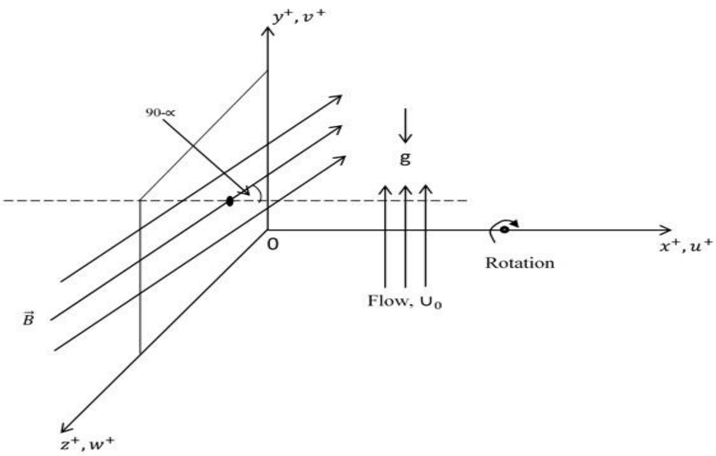
II. Model Formulation

An unsteady incompressible Magnetohydrodynamics boundary flow of an electrically conducting fluid past an impulsively started vertical plate is considered. A strong, steady and inclined magnetic field of strength $\vec{B}$ is applied into the fluid region at an angle $\propto$ to the plate. The plate and the fluid are in a state of rigid rotation with uniform angular velocity Ω about the x^+-axis. The y^+-axis is taken along the plate in the vertically upward direction. The plate and the fluid are assumed to be at the same initial temperature $T^+{}_\infty$ and concentration $C^+{}_\infty$ everywhere. At $t^+ > 0$, the plate is exponentially accelerated with velocity $u^+ = \frac{U_0}{A} e^{a^+ t^+}$ in its own plane in the vertical direction against the direction of gravitational field. Simultaneously

Its temperature and concentration are raised to $T^+{}_w$ and $C^+{}_w$ respectively and there after maintained constant.

Fig. 1: Flow Configuration

The dimensional model equations governing the flow are:



$$\frac{\partial v^+}{\partial t^+} - 2\Omega^+ w^+ = \upsilon \frac{\partial^2 v^+}{\partial x^{+2}} + g\beta(T^+ - T_\infty{}^+) + g\beta^*\left(C^+ - C_\infty{}^+\right) + \frac{\sigma \mu_e{}^2 \lambda^2 H_o{}^2 (m^* \lambda w^+ - v^+)}{\rho(1 + m^{*2}\lambda^2)} \quad (1)$$

$$\frac{\partial w^+}{\partial t^+} + 2\Omega^+ v^+ = \upsilon \frac{\partial^2 w^+}{\partial x^{+2}} - \frac{\sigma \mu_e{}^2 \lambda^2 H_o{}^2 (m^* \lambda v^+ + w^+)}{\rho(1 + m^{*2}\lambda^2)} \quad (2)$$

$$\frac{\partial T^+}{\partial t^+} \frac{k}{\rho c_p} \frac{\partial^2 T^+}{\partial x^{+2}} + \frac{\upsilon}{C_p}\left[\left(\frac{\partial v^+}{\partial x^+}\right)^2 + \left(\frac{\partial w^+}{\partial x^+}\right)^2\right] + \frac{\sigma \mu_e{}^2 \lambda^2 H_o{}^2 (v^{+2} + w^{+2})}{\rho C_p (1 + m^{*2}\lambda^2)} \quad (3)$$

$$\frac{\partial C^+}{\partial t^+} = D \frac{\partial^2 C^+}{\partial x^{+2}} \quad (4)$$

The dimensional initial and boundary conditions governing the flow are:

$w^+ = 0,\ v^+ = 0,\ T^+ = T^+_\infty,\ C^+ = C^+_\infty$ at $t^+ < 0$, for all x^+

$w^+ = 0,\ v^+ = \frac{U_o}{A} e^{a^+ t^+},\ T^+ = T^+_w, C^+ = C^+_w$ for all $t^+ > 0,\ x^+ = 0$ (5)

$w^+ \to 0, v^+ \to 0,\ T^+ \to T^+_\infty, C^+ \to C^+_\infty$ as $x^+ \to \infty$,

where $A = \sqrt[3]{\frac{U_0^2}{\nu}}$ is a constant.

III. Non-dimensionalization

This is achieved using the dimensionless quantities below:

$$v = \frac{v^+}{\sqrt[3]{U_0 \nu}},\quad w = \frac{w^+}{\sqrt[3]{U_0 \nu}},\quad t = t^+ \sqrt[3]{\frac{U_0}{\nu}},$$

$$\Omega = \Omega^+ \sqrt[3]{\frac{\nu}{U_0^2}},\quad M^2 = \frac{\sigma \mu_e^2 H_o^2}{2\rho} \sqrt[3]{\frac{\nu}{U_0^2}}$$

$$x = x^+ \sqrt[3]{\frac{U_0}{\nu^2}},\quad a = a^+ \sqrt[3]{\frac{\nu}{U_0^2}},$$

$$Gr = \frac{g\beta(T_w^+ - T_\infty^+)}{U_0},\ \theta = \frac{T^+ - T_\infty^+}{T_w^+ - T_\infty^+},\ Sc = \frac{\upsilon}{D}$$

$$Ec = \frac{\sqrt[3]{U_0^2}\nu^2}{C_p(T_w^+ - T_\infty^+)},\quad Pr = \frac{\upsilon \rho C_p}{k} = \frac{\mu C_p}{k} \quad (6)$$

$$Gc = \frac{g\beta^*(C_w^+ - C_\infty^+)}{U_0},\quad C = \frac{C^+ - C_\infty^+}{C_w^+ - C_\infty^+},$$

The dimensionless model equations become:

$$\frac{\partial v}{\partial t} - 2\Omega w = \frac{\partial^2 v}{\partial x^2} + Gr\theta + GcC + \frac{2M^2\lambda^2}{1+m^{*2}\lambda^2}(m^*\lambda w - v) \quad (7)$$

$$\frac{\partial w}{\partial t} + 2\Omega v = \frac{\partial^2 w}{\partial x^2} - \frac{2M^2\lambda^2}{1+m^{*2}\lambda^2}.(m^*\lambda v + w) \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{Pr\,\partial x^2} + Ec\left[\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] + \frac{2M^2\lambda^2 Ec}{1+m^{*2}\lambda^2}.(v^2+w^2) \quad (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial x^2} \quad (10)$$

Subject to the initial and boundary conditions:

$w = 0, v = 0,\ \theta = 0,\ C = 0$ at $t < 0$, for all x

$w = 0,\ v = e^{at},\ \theta = 1, C = 1$ for all $t > 0,\ x = 0$ (11)

$w \to 0,\ v \to 0,\ \theta \to 0, C \to 0$ as $x \to \infty$.

The dimensionless model partial differential equations obtained are highly non-linear. The equations are written in finite difference form to become:

$$v(i,j+1) = \frac{\Delta t}{(\Delta x)^2}[v(i-1,j) - 2v(i,j) + v(i+1,j)] + \Delta t[Gr\theta(i,j) + GcC(i,j) + 2\Omega w(i,j)] + \frac{\Delta t M^2\lambda^2}{1+m^{*2}\lambda^2}[m^*\lambda w(i,j) - v(i,j)] \quad (12)$$

$$w(i,j+1) = \frac{\Delta t}{(\Delta x)^2}[w(i-1,j) - 2w(i,j) + w(i+1,j)] - \left[\frac{m^* M^2\lambda^3}{1+m^{*2}\lambda^2} + 2\Omega\right]\Delta t v(i,j) - \left[\frac{\Delta t M^2\lambda^2}{1+m^{*2}\lambda^2} - 1\right]w(i,j) \quad (13$$

$$\theta(i,j+1) = \frac{\Delta t}{Pr(\Delta x)^2}[\theta(i-1,j) - 2\theta(i,j) + \theta(i+1,j)] + Ec\frac{\Delta t}{(\Delta x)^2}[\{v(i+1,j) - v(i,j)\}^2 + \{w(i+1,j) - w(i,j)\}^2] + Ec\frac{\Delta t M^2\lambda^2}{1+m^{*2}\lambda^2}[v^2(i,j) + w^2(i,j)] + \theta(i,j) \quad (14)$$

$$C(i,j+1) = \frac{\Delta t}{Sc(\Delta x)^2}[C(i-1,j) - 2C(i,j) + C(i+1,j)] + C(i,j). \quad (15)$$

Subject to the initial conditions

$$\left.\begin{matrix} q = 0 \\ \theta = 0 \\ C = 0 \end{matrix}\right\} j \le 0 \quad (16)$$

And boundary conditions

$$\left.\begin{matrix} q = 1 \\ \theta = 1 \\ C = 1 \end{matrix}\right\} j > 0 \quad (17)$$

$q \to 0, \theta \to 0, C \to 0$ as $j \to \infty$.

IV. Results and Discussion

To understand the effects of Hall current, rotation and angle of inclination of the applied magnetic field on the primary velocity, secondary velocity, temperature and concentration of the fluid, the following parameter values were used: $Sc = 0.6$, $Ec = 0.02$, $Gr = -5$, $Pr = 0.71$, $Gc = 1$ and $M^2 = 5$. The results were represented in graphs as in fig. 2 to fig. 13.

From fig. 2, increase in rotational parameter lowers the primary velocity. The primary velocity is maximum at the plate due to the no-slip condition.

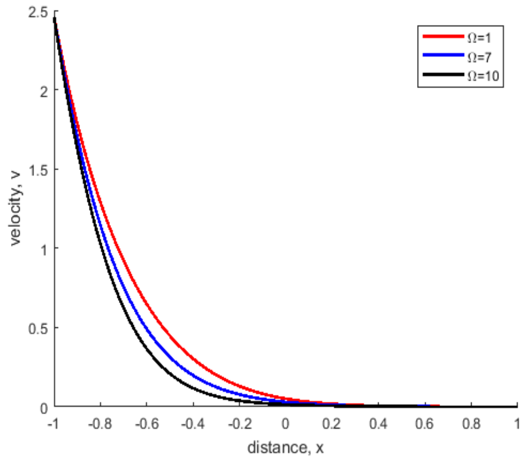


Fig. 2: Primary velocity profiles for different Ω.

Fig.3 shows the effect of the rotation parameter on the transverse (secondary) velocity. It is observed that as the rotation parameter is increased, the secondary velocity increases.

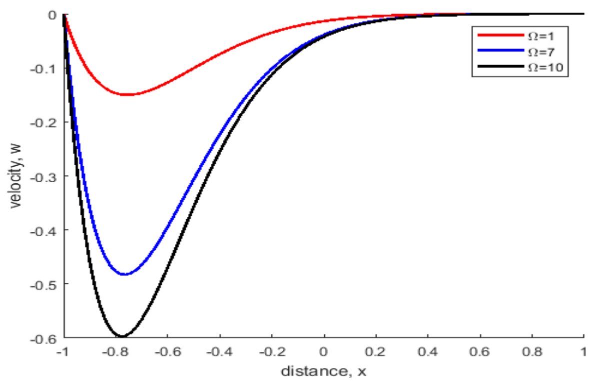


Fig. 3: Secondary velocity profiles for different Ω.

Fig.4 shows rotation has no effect on the temperature profile of the fluid.

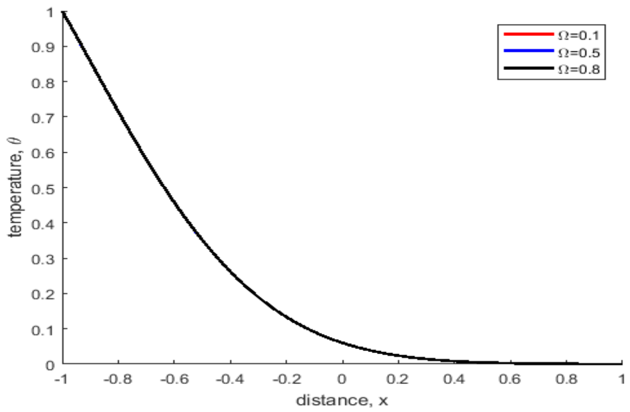


Fig.4: Temperature profiles for different values of Ω.

Fig.5 show that rotation parameter has no effect on the fluid concentration.

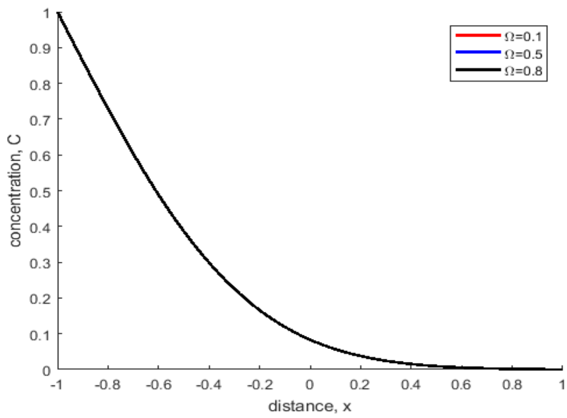


Fig. 5: Concentration profiles for different Ω.

Fig. 6 illustrates the effects inclining the applied magnetic field has on the axial (primary) velocity. Increasing the angle of inclination of the applied magnetic field lowers the axial velocity profiles. The effect is however meager.

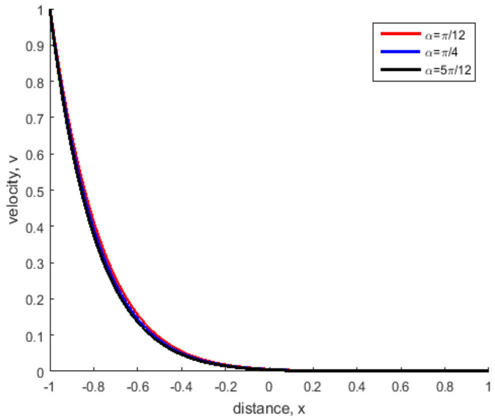


Fig. 6: Primary velocity profiles for different α.

Fig. 7 illustrates the effects inclination of the applied magnetic field has on the transverse velocity. Increasing the angle of inclination of the applied magnetic field raises the transverse velocity profiles. The effect is however weak. At the plate surface, transverse velocity is zero. The velocity increases to a maximum value away from the plate when $x=-0.8$ then decreases to a constant zero value far away from the plate.

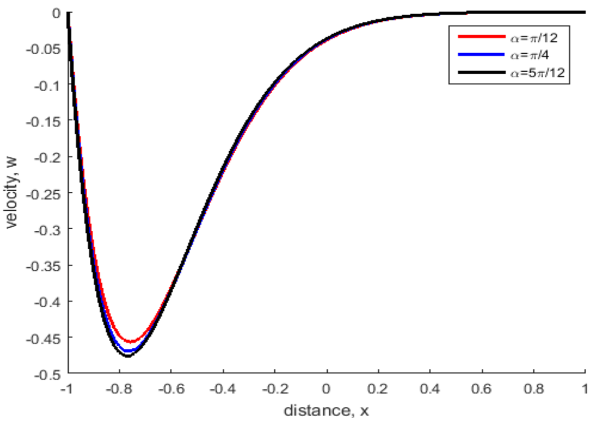


Fig.7: Secondary velocity profiles for different α.

From fig. 8, the angle of inclination of the magnetic field has no effect on the temperature of the fluid.

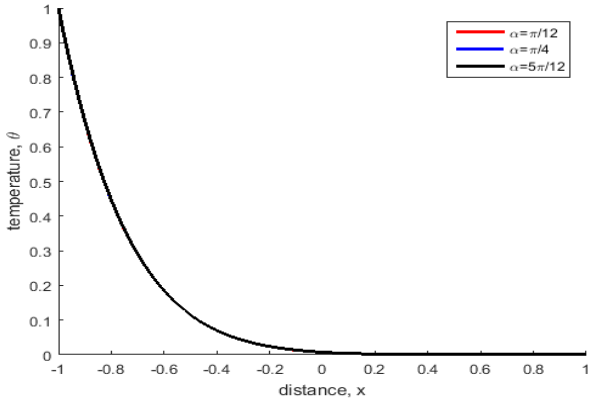


Fig.8: Temperature profiles for different values of α.

Fig.9 demonstrates the effect of the angle of inclination of the applied magnetic field on the concentration of the fluid. The graph shows that variation of the angle of inclination of magnetic field has no effect on the fluid concentration.

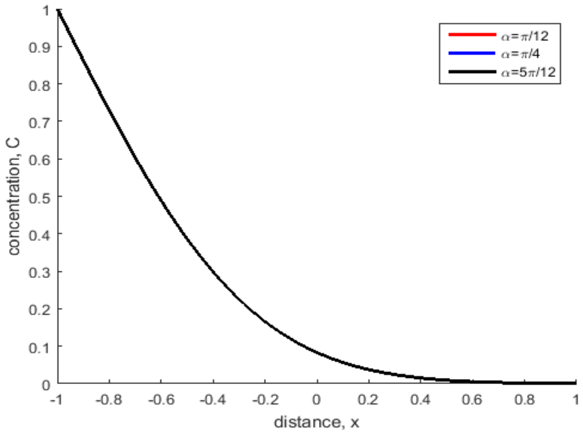


Fig. 9: Concentration profiles for different α.

Fig. 10 illustrates the effects of the Hall current parameter on the axial velocity. Change in the Hall current parameter does not produce any significant change in the primary velocity. The primary velocity is maximum at the surface of the plate and decreases uniformly to zero far away from the plate.

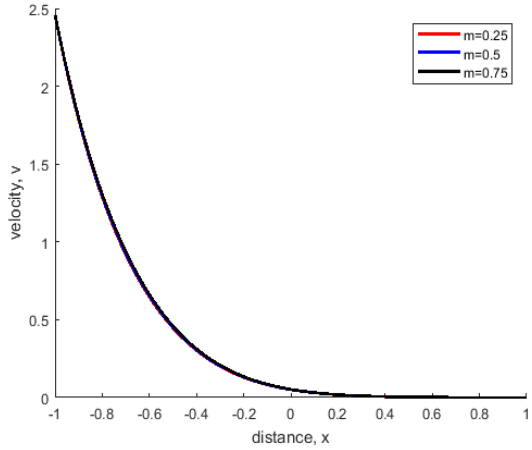


Fig. 10: Primary velocity profiles for different m*.

Secondary velocity profiles for various values of the Hall current parameter are shown in Fig. 11. It is observed that the secondary velocity reduces as the Hall current parameter is increased.

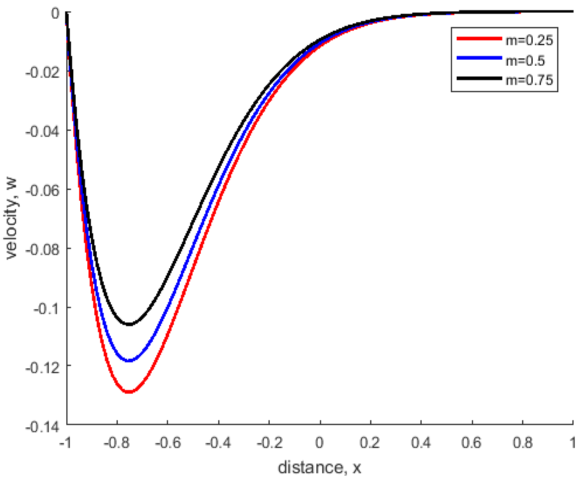


Fig.11: Secondary velocity profiles for different m*.

Temperature profiles for various values of the Hall current parameter are shown in Fig. 12. Variation in the Hall current parameter does not affect the temperature of the fluid medium.

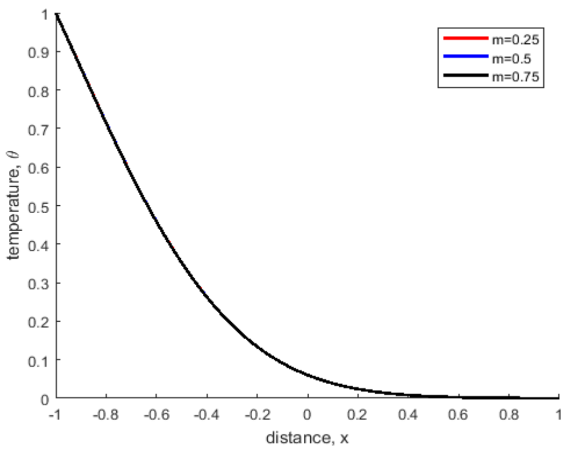


Fig. 12: Temperature profiles for different m*.

The effects of the Hall current parameter on the concentration of the fluid medium is shown in Fig. 13. The Hall parameter has no effect on the concentration of the fluid. Maximum fluid concentration occurs at the surface of the plate. The concentration reduces uniformly to a minimum far away from the plate surface.

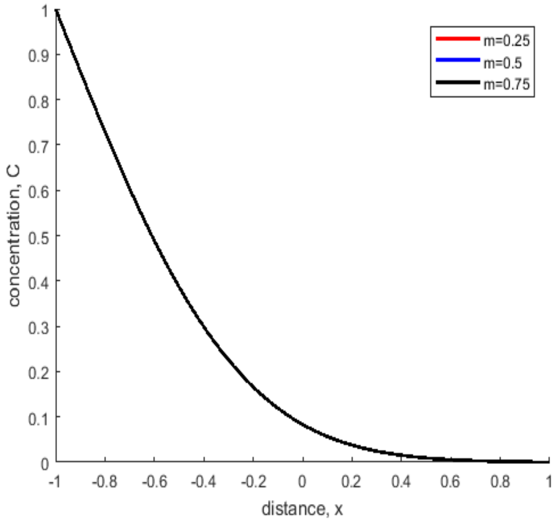


Fig. 13: Concentration profiles for different m*.

V. Conclusion

An increase in the angle of inclination of the applied magnetic field has no effect on the temperature and concentration of the fluid. It however slows down the primary velocity while it increases the secondary velocity of the fluid. The Hall current parameter has no effect on the primary velocity, fluid temperature or even fluid concentration. It however reduces the secondary velocity of the fluid. When the rotation parameter is raised, the primary velocity is reduced while the secondary velocity is increased. Fluid temperature and concentration are however not influenced by the rotation parameter.

The skin friction and the rate of heat transfer of the fluid depend on the angle of inclination of the magnetic field, the rotation parameter and the Hall current parameter. In particular, the rate of heat transfer increases with increase in the Hall current parameter while it decreases with increase in the angle of inclination of the applied magnetic field and the rotation parameter. The magnitudes of the stresses τ_x and τ_y reduces with increase in the Hall current parameter.

VI. References

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SYMBOLS AND INDEX OF NOTATIONS

$\vec{i}, \vec{j}, \vec{k}$	Unit vectors along x, y and z respectively
$(\vec{u}, \vec{v}, \vec{w})$	Dimensionless velocity components
$(\vec{u}^+, \vec{v}^+, \vec{w}^+)$	Dimensional velocity components
t	Dimensionless time
t^+	Dimensional time
θ	Dimensionless temperature
θ^+	Dimensional temperature
T_w^+	Wall temperature
T_∞^+	Fluid temperature at infinity
(x, y, z)	Cartesian Co-ordinates
T	Dimensionless temperature
T_∞	Dimensionless free stream temperature
$\vec{U}_\infty$	Free stream velocity
h	Heat transfer coefficient
$\vec{J}$	Current density
e	Electric Charge
$\vec{g}$	Acceleration due to gravity
C_p	Specific heat Capacity at constant pressure
σ	Electrical conductivity of the fluid
k	Coefficient of thermal conductivity
υ	Kinematic Coefficient of viscosity
ρ	Fluid density
$\vec{q}$	Fluid velocity
α	Angle of the applied magnetic field
λ	Cosine of the angle of the applied magnetic field
Gr	Grashof number
Pr	Prandtl number
Ec	Eckert number

μ_e	Magnetic permeability of the medium
μ	Coefficient of viscosity
$\vec{B}$	Applied Magnetic field
$\vec{E}$	Electric field
M	Magnetic field parameter
m^*	Hall Current parameter
Q	Charge per unit volume
Ω	Angular velocity of the fluid
Sh	Sherwood number
Nu	Nusselt number
β	Volumetric coefficient of thermal expansion
β^*	Volumetric coefficient of thermal expansion due to concentration gradient
C	Dimensionless fluid concentration
C_w^+	Concentration at the plate
C_∞^+	Concentration far from the plate