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Effect of Prandtl Number on Magneto-Convection in a Lid Driven Square Cavity with a Sinusoidal Vertical Wall

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***Abstract*— In the present study the effect of Prandtl number on magneto-convection in a lid driven square cavity with a sinusoidal vertical wall were investigated numerically. The horizontal bottom and top walls are adiabatic. The left and right vertical walls are temperature T_h and T_c respectively with $T_h>T_c$. The governing equations along with appropriate boundary conditions for the present problem are first transformed into a non-dimensional form and the resulting non linear system of partial differential equations are then solved numerically using Galerkin's finite element method. Parametric studies of the fluid flow and heat transfer in the enclosure are performed for Prandtle number Pr, Reynolds number Re. and sinusoidal λ. The streamlines, isotherms, average Nusselt number at the hot wall and average temperature of the fluid in the enclosure are presented. The numerical results indicate that the Reynolds number has strong influence on the streamlines and isotherms. On the other hand, Prandtl number and undulation λ have little effect on the stream line and isotherm plots. Finally, the mentioned parameters have significant effect on average Nusselt number at the hot wall and average temperature of the fluid in the enclosure.**

Keywords—Prandtlnumber, Magnetohydrodynamic, Finite element method, Mixed convection and Lid driven enclosure

I. INTRODUCTION

Mixed convection flow and heat transfer in lid-driven cavities occurs as a result of two competing mechanisms. The first is due to shear flow caused by the movement of one of the walls in the enclosure, while the second is due to buoyancy flow produced by thermal non homogeneity of the enclosure boundaries. Analysis of mixed convective flow in a lid driven enclosure finds applications in materials processing, flow and heat transfer in solar ponds, dynamics of lakes, reservoirs and cooling ponds, crystal growing, float glass production, metal casting, food processing, galvanizing, and metal coating, among others. Many authors have recently studied heat transfer in enclosures with partitions, which influence the convection flow phenomenon. Aydin (1999) conducted a numerical study to investigate the transport mechanism of laminar mixed convection in a shear and buoyancy driven cavity. Two orientations of thermal boundary conditions at the cavity walls were considered to simulate the aiding and opposing buoyancy mechanism. Rahman et al. (2009) conducted finite element analysis of mixed convection in a rectangular cavity with a heat-conducting horizontal circular cylinder. The present study demonstrates the capability of the finite element formulation that can provide insight to steady-state incompressible conjugate effect of mixed convection and conduction problem. Saha et al. (2010) studied numerically steady state two-dimensional mixed convection problem in a square enclosure where they observed increasing heat transfer rate with dominant internal heat generation. Nasrin and Parvin (2011) analyzed the hydrodynamic effect on mixed convection in a lid driven cavity with sinusoidal wavy bottom surface. They observed the highest heat transfer rate at the lowest magnetic effect. Rabienataj Darzi et al' (2011) investigated mixed convection simulation of inclined lid driven cavity using lattice Boltzmann method. They found laminar mixed convection for three Richardson numbers that present forced convection dominating, mixed convection and natural convection dominating are investigated using lattice Boltzmann method for various inclination angles of lid-driven cavity. Finite element analysis of magneto-hydrodynamic MHD mixed convection flow on a triangular cavity was formulated by Akhi Farhana et al. (2011). Parvin and Hossain investigated on the conjugate effect of joule heating and magnetic field on combined convection in a lid-driven cavity with undulated bottom surface where they decided that the increase in the Hartmann number hinders the flow and consequently the isothermal lines occupy almost the entire region of the cavity and the variation in the Joule heating parameter affects significantly the flow and thermal current activities. Dawood and Teamah (2012) performed hydro-magnetic mixed convection double diffusive in a lid driven square cavity. Salam Hadi Hussain (2013) analyzed magnetohydrodynamics opposing mixed convection in two-sided lid-driven differentially heated parallelogrammic cavity. Hydro-magnetic mixed convection flow in a lid-driven cavity with wavy bottom surface was conducted by Saha et al. (2014) where they fund that the variation in the Reynolds number affects significantly the flow and thermal current activities. The increase in the ratio of Grashof number and square of Reynolds number (Richardson number) to obstruct flow and thermal current activities owing to the increase in the imposed vertical temperature gradient. Hussein & Hussein (2015) studied characteristics of magnetohydrodynamic mixed convection in a parallel motion two-sided lid- driven differentially heated parallelogrammic cavity with various skew angles. Recently again Saha et al. (2015) analyzed the effect of internal heat generation or absorption on MHD mixed convection flow in a lid driven cavity. Heat transfer rate decreases with increasing of Hartmann number and heat generation parameter where as increases for the increasing values of heat absorption parameter. Thus, magnetic field plays an important role to control heat transfer and fluid flow. Very recently Malleswaran and Sivasan Karan (2016) investigated numerically MHD mixed convection in a lid-driven cavity with corner heaters. They concluded cavity with corner heaters is completely different from differentially heated cavity in which the thermal boundary layer occurs near both hot and cold walls whereas no such boundary layer exist in the cavity with corner heaters at forced convection mode.

To the best of the author's considerate, little attention has been paid to the problem effect of Prandtl number on magneto-convection in a lid driven square cavity with a sinusoidal vertical wall. The objective of the present study is to examine the momentum and energy transport processes in a lid-driven cavity of wavy surface with different undulation.

II. . PROBLEM FORMULATION

The present problem is a two-dimensional square cavity with a side length L. The physical system considered in the present study is displayed in Fig.1.The top and bottom walls are taken adiabatic and impermeable while the vertical walls are maintained at uniform but different temperatures such that the right wall is assigned to temperature Tc while the left wall is subjected to temperature T_h under all circumstances T_h > Tc condition is maintained. Furthermore, the right wall is assumed to slide from bottom to top at a constant speed U_0 and left wall is sinusoidal wavy pattern A magnetic field of strength B_0 is acting in a transverse direction normal to the side walls.

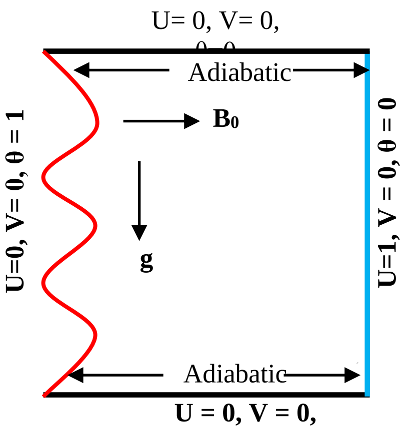


Fig.1. Schematic diagram of the cavity

III. . MATHEMATICAL FORMULATION

The functioning fluid is assumed to be Newtonian and incompressible with the flow is set to operate in the laminar mixed convection regime. The leading equations under Boussinesq approximation in dimensionless type are as follows:

Continuity Equation

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

Momentum Equations

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)$$
$$+\frac{Ra}{\mathrm{Re}^2\,\mathrm{Pr}}\theta-\frac{Ha^2}{\mathrm{Re}}V \tag{3}$$

Energy Equations

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+JV^2 \tag{4}$$

The dimensionless variables are defined as:

$$X=\frac{x}{L},Y=\frac{y}{L},U=\frac{u}{U_0},V=\frac{v}{U_0},$$
$$P=\frac{p}{\rho U_0^2},\theta=\frac{T-T_c}{T_h-T_c} \tag{5}$$

where the dimensionless quantities *x* and *y* are the coordinates varying along horizontal and vertical directions respectively, *u* and *v* are the velocity components along the *x* and *y* axes respectively, θ is the temperature of fluid and *p* is the pressure.

The governing parameters in the preceding equations are the Reynolds number R*e*, Hartmann number H*a*, Joule heating parameter J and Prandtl number P*r*.

$$\mathrm{Re}=\frac{U_0L}{\nu},\ Ra=\frac{g\beta\left(T_h-T_c\right)L^3}{\nu\alpha},\ \mathrm{Pr}=\frac{\nu}{\alpha},$$ are

$$J=\frac{\nu\beta_0^2U_0L}{\rho C_p\left(T_h-T_c\right)}$$

Reynolds number, Rayleigh number, Prandtl number and Joule heating parameter respectively and *Ha* is the Hartmann number which is defined as $Ha^2=\frac{\sigma B_0^2L^3}{\mu}$ the shape of the vertical corrugated wavy surface profile is assumed to be mimic the following pattern $X=A\sin\left(2\lambda\pi Y\right)$ where a dimensionless amplitude of the wavy surface and λ is the number of undulations. The dimensionless boundary conditions of the present problem are as follows: at the sliding lid $U=1,V=0,\theta=0$; at the horizontal walls: $U=0, V=0, \frac{\partial\theta}{\partial N}=0$ and at the left vertical wavy wall $U=0,\ V=0,\ \theta=1$.the rate of heat transfer is computed at the wavy wall and is expressed in terms of the local Nusselt number $\overline{Nu}$ as

$$\overline{Nu}=\frac{hL}{K}=-\frac{\partial\theta}{\partial N}L$$

where, N represents the coordinate direction normal to the surface. The dimensionless normal temperature gradient can be written as

$$\frac{\partial\theta}{\partial N}=-\frac{1}{L}\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}$$

while the average Nnusselt number (nu) is obtained by integrating the local Nnusselt number along the vertical wavy surface and is defined by $Nu=\frac{1}{s}\int_0^s \overline{Nu}\,dn$

where s is the total chord length of the wavy surface. The average temperature of the fluid in the enclosure is defined by $\theta_{av}=\int \theta d\bar{V}/\bar{V}$ and $V_{av}=\int V d\bar{V}/\bar{V}$ is the velocity, where V is the magnitude of the velocity and $\bar{V}$ is the cavity volume.

IV. METHOD OF SOLUTION

Rahman et al. [2009b] employed to investigate the mixed convection in an obstructed lid-driven cavity. In this method, the continuum domain is divided into a set of non-overlapping regions called elements. Six node triangular elements with quadratic interpolation functions for velocity as well as temperature and linear interpolation functions for pressure are utilized to discretize the physical domain. Moreover, interpolation functions in terms of local normalized element coordinates are employed to approximate the dependent variables within each element. Substitution of the obtained approximations into the system of the governing equations and boundary conditions yield a residual for each of the conservation equations. These residuals are reduced to zero in a weighted sense over each element volume using the Galerkin method.

The velocity and thermal energy equations result in a set of non-linear coupled equations for which an iterative scheme is adopted. The application of this technique and the discretization procedures are well documented by Taylor and Hood [1973] and Dechaumphai [1999]. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ε such that

$$\sum\left|\phi_{ij}^{m}-\phi_{ij}^{m-1}\right|\leq\varepsilon,$$

where Ø represents a dependent variable U, V, P, and θ, the indexes i, j indicate a grid point, and the index m is the current iteration at the grid level. The convergence criterion was set to 10^{-5}.

V. GRID REFINEMENT TEST

In order to determine the proper grid size for this study, a grid independence test is conducted with five types of mesh for Ha = 20, Pr = 0.71, Ra = 104, A = 0.05, Re = 100 and λ= 3 which is shown in Table 1. The extreme value of Nu is used as a sensitivity measure of the accuracy of the solution and is selected as the monitoring variable. Considering both the accuracy of numerical values and computational time, the present calculations are performed with 37123 nodes and 5604 elements grid system.

Table 1: Grid Sensitivity Check at Ha = 20, J =1, Pr = 0.71, Ra = 10^4, Re = 100 and λ = 3

Nodes Elements	4931 518	10853 1632	18916 2832	37123 5604	47853 7244
Nu	0.99397	3’026590	3.048085	3.050462	3.050465
θ_{av}	0.51481	1.071636	1.081563	1.091666	1.09166

VI. RESULTS AND DISCUSSION

The characteristics of the flow and temperature fields in the lid-driven cavity are examined by exploring the effects of the Prandtl number Pr, Reynolds number Re along with number of undulations λ. Such field variables analyzed by outlaying the steady state version of the velocity, streamline and temperature distributions as well as the average Nusselt number Nu and maximum temperature θ_{av}. In the current numerical investigation, the following parametric domains of the dimensionless groups are considered $0.1\leq Re\leq 10$, $0.071\leq Pr\leq 10$, Ra = 10000, Ha = 20 and J =1.0. The impact of varying the Prandtle number Pr on the streamlines for different undulation gauged the results illustrated in Figures 2. A circulating clockwise cell that tries to occupy the bulk of the enclosure is seen. For lower values (Pr = 0.071and 0.71) of Prandtl number thermal diffusivity dominates, this means that conduction dominates over convection. On the other hand for higher values of Prandtl number convection dominates over conduction. The streamlines has tendency to cover the whole cavity in absence of undulations distributed. And for λ = 1, 2 and 3 the circulatory flow moves upwards and becomes smaller in size in the cavity.

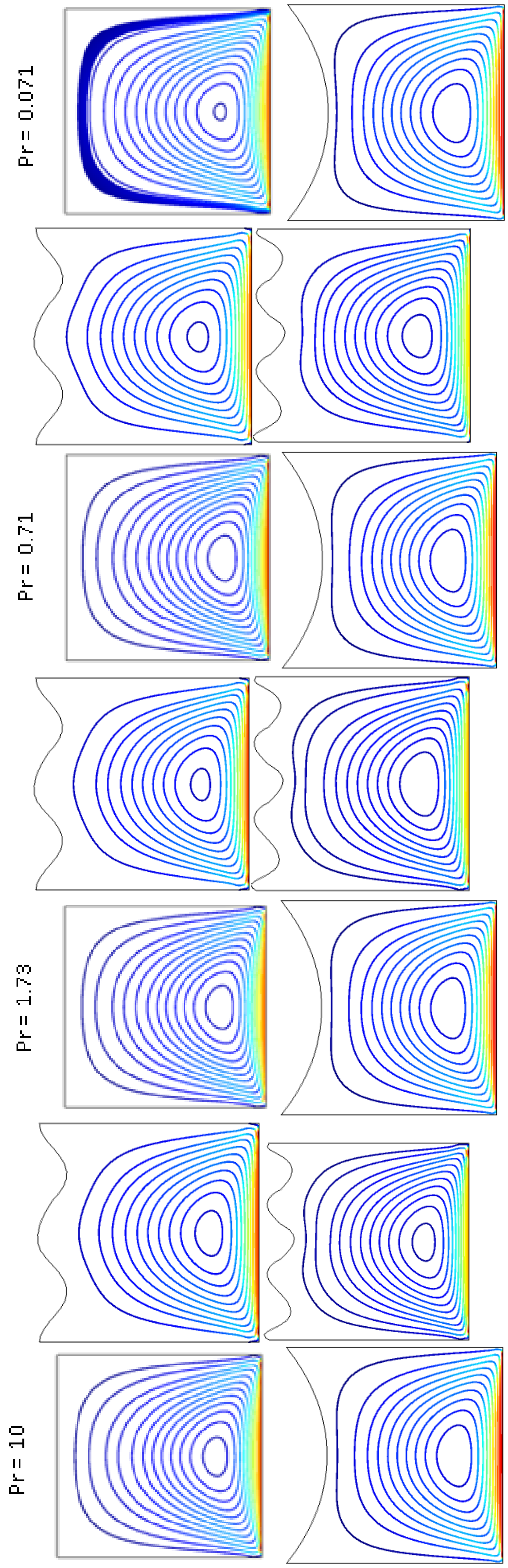
For low Pr inertia effect is more significant. For a particular Pr, changes in streamlines are noticeable to the variation of λ. Figure.3 shows the corresponding temperature field for variation Pr and λ at Re = 50, Ra = 10000, Ha = 20 and J = 1.0. For fixed λ the isothermal lines changes their pattern with varying Pr. Thermal boundary layer thickness and the thermal lines are condensed at the adjacent area of the heated surface as Pr increases. High temperature gradient is seen for highly viscous fluid with large Prandtl number.

The variation local Nusselt number, temperature and velocity at the horizontal center line of the fluid for different Prandtl number and λ have been presented in Figure 6, Figure5 and Figure 4 respectively. The local Nusselt number is highest and velocity of the fluid is lowest for the largest value of Pr (=10). This is because the fluid with the highest Prandt number is capable to carry more heat away from heat source through dominant convection. At large Pr, fluids viscosity becomes very high which retards the velocity of the fluid. Figure 4 represents the velocity graph for varying of Prandtl number. It is seen from the figure that for highest values of Prandtl number lowest and highest velocity at a certain point. In addition λ has less significant effect on velocity, temperature and local Nusselt number.

The impact of varying the Reynolds number on the streamlines for various numbers of undulations is gauged through the results illustrated in figure7. The Reynolds number provides a measure for mechanically induced lid-driven force convection effect. Again, the consideration domain Re was varied from 0.1 to 10 which very mach spans over the range of possible operating conditions. For various Re the overall features of the streamline are similar to those of conventional mechanically-driven cavity flow which characterized circulating clockwise vortex that occupies about the bulk of the cavity. For lower values of Reynolds number streamlines are distributed in the whole cavity except centre and creates a second vortex at the corner of the cavity. But at higher Reynolds number these lines moves towards the center from the boundary and second vortex becomes absent. These phenomena are seen all over the all vales of wave number. At low Reynolds number the viscous force becomes more significant due to create a secondary vortex. However, for a particular Reynolds number, variation in λ causes significant changes in streamlines. With the increasing values of λ cavity core fills with less streamlines. On the other hand, corresponding temperature field is shown in Figure 8. For a fixed value of λ, the thermal field has little change with the variation of in Re. Moreover, the thermal boundary layer near the wavy surface becomes less concentrated when the wave increases. Isothermal lines take the wavy pattern due to waviness of the surface.

Figure 9, Figure 10 and Figure 11depict the velocity, temperature and local Nusselt number respectively of the fluid for various Re and .λ. Local Nusselt number is highest for lowest values of Reynolds number at λ = 0. On the other hand local Nnusselt number is observed lowest and highest for lowest values of Reynolds number Re at λ = 1, 2 and 3 while local Nusselt number decreases increasing of Reynolds number (except Re = 0.1) for all wavy surface. This happens because viscosity dominates lowest values of Reynold number highest temperature is observed for all λ.

No significant changes is observed in temperature for Reynolds number (= .7, 2 and 10) for all λ. Highest and lowest velocity is observed at a certain point for decreasing Reynolds number at al wavy surface.



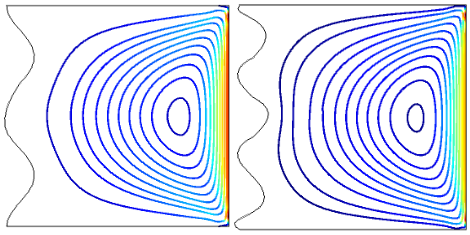
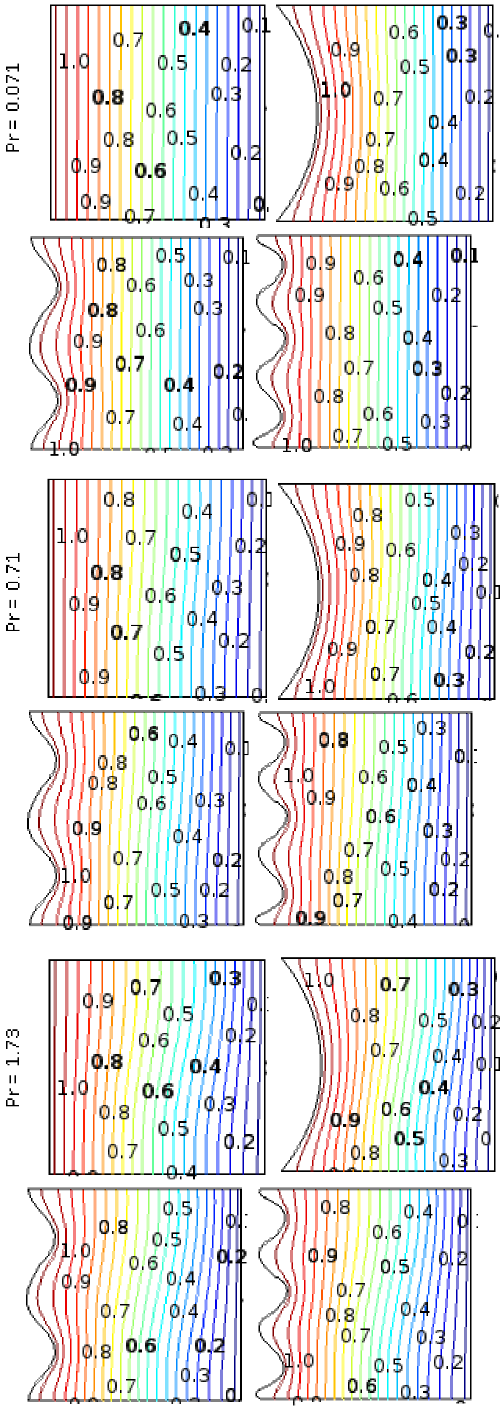


Fig.2. streamlines for variation *Pr* and λ at *Re* = 50, *Ra* = 10000, *Ha* = 20 and *J* = 1.0.



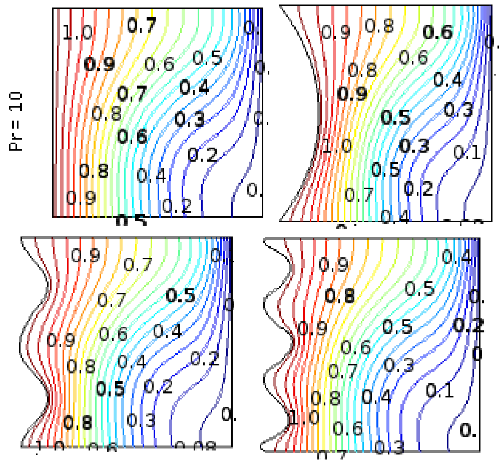


Fig.3. isotherms for variation *Pr* and λ at *Re* = 50, *Ra* = 10000, *Ha* = 20and *J* = 1.0

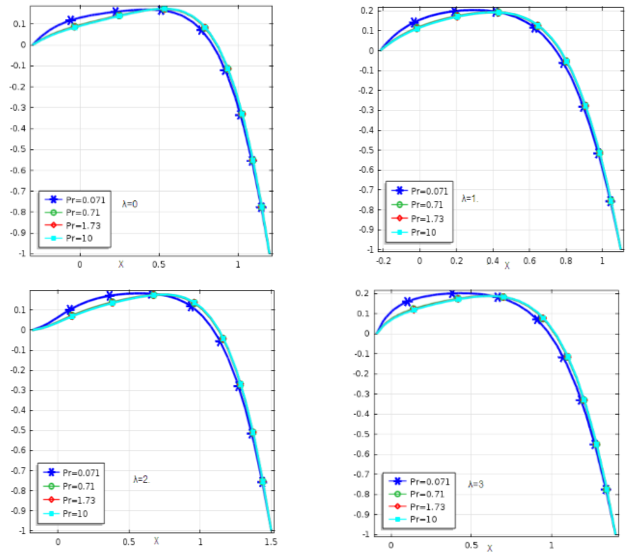


Fig.4. Velocity graph for variation *Pr* and λ = 0, 1, 2, and 3 at *Re* = 50, *Ra* = 1000, *Ha* = 20, *Re* = 1 and *J* = 1.0.

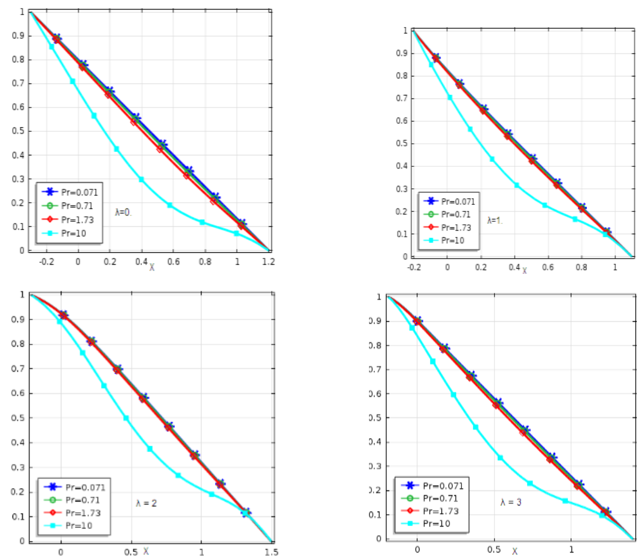


Fig.5. Temperature graph for variation *Pr* and λ = 0, 1, 2, and 3 at *Ra* = 10000, *Pr* = 0.71, *Ha* = 20, *Re* = 1 and *J* = 1.0

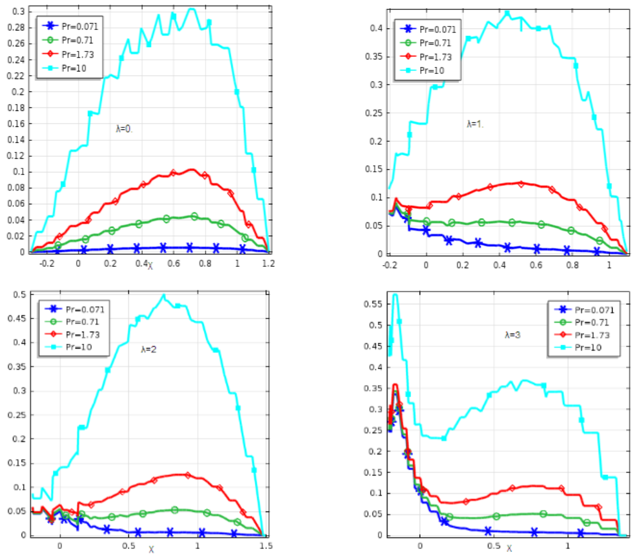
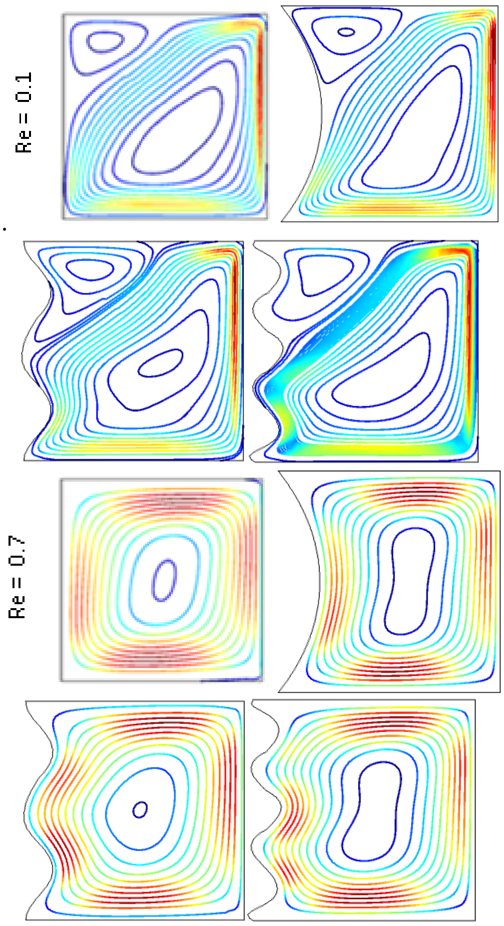


Fig.6. Variation of Local Nusselt number for various values of *Pr* and λ = 0, 1, 2and 3 at *Ra* = 10000, *Ha* = 20, *Re* = 1and *J* =1.0



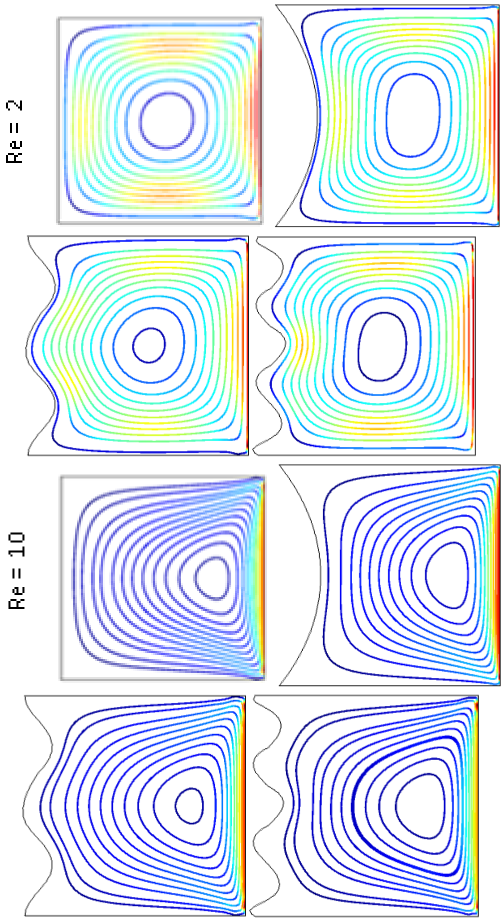
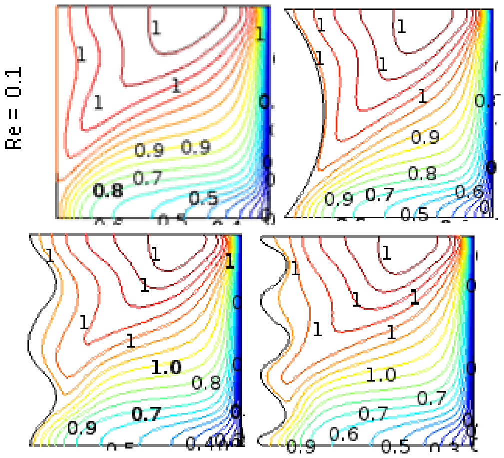


Fig.7. Streamlines for variation *Re* and λ at *Ra* = 10000, *Pr* = 0.71, *Ha* = 20 and *J* = 1.0.



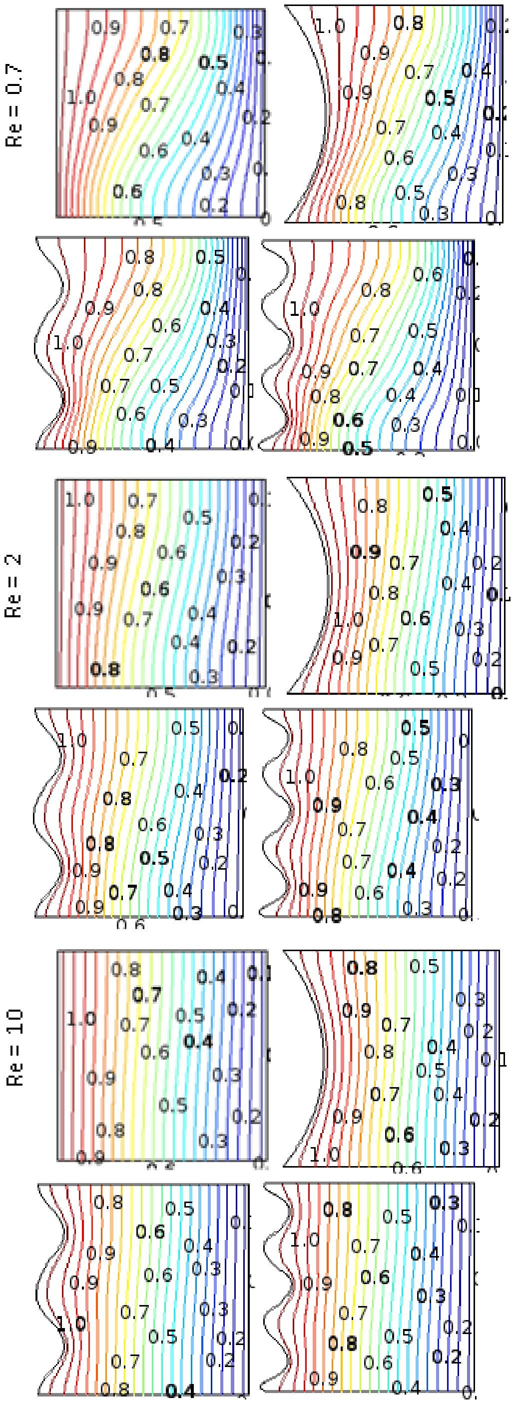
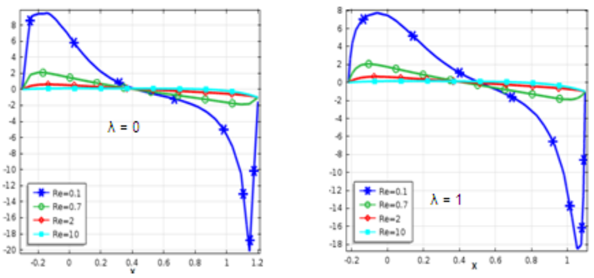


Fig.8. Isotherms for variation *Re* and λ at *Ra* = 10000, *Pr* = 0.71, *Ha* = 20 and *J* = 1.0



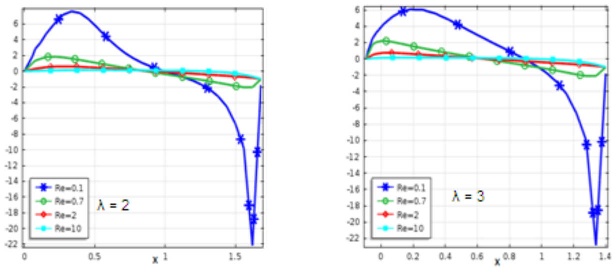


Fig.9. Velocity graph for variation *Re* and λ = 0, 1, 2, and 3 at *Ra* = 10000, *Pr* = 0.71, *Ha* = 20 and *J* = 1.0

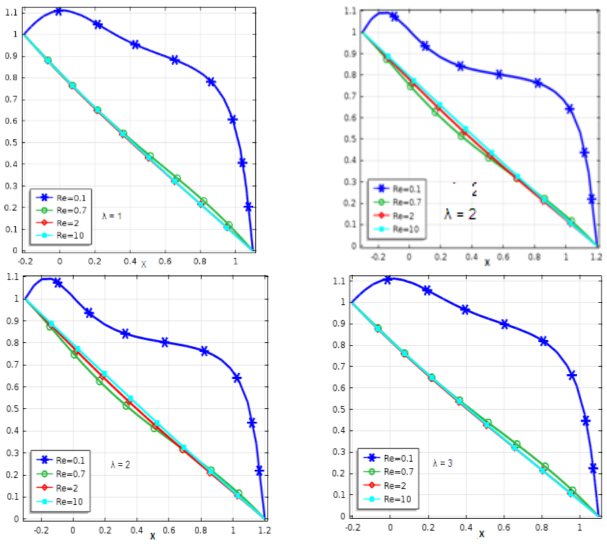


Fig.10. Temperature graph for variation *Re* and λ = 0, 1, 2, and 3 at *Ra* = 10000, *Pr* = 0.71, *Ha* = 20 and *J* = 1.0

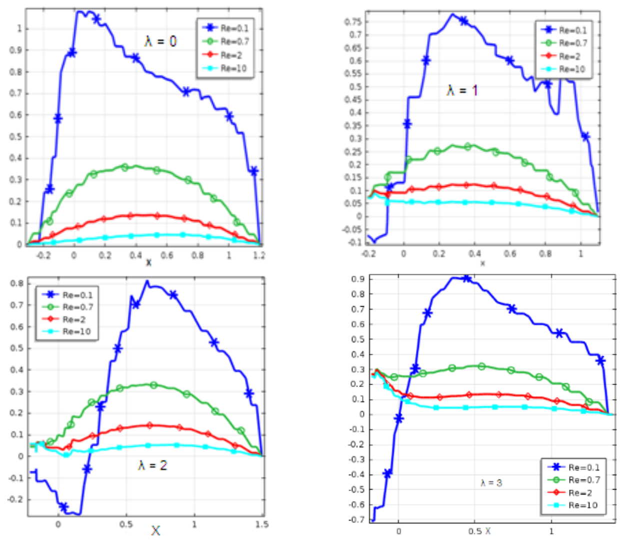


Fig.11. Variation of Local Nusselt number for various values of *Re* and λ = 0, 1, 2and 3 at *Ra* = 10000, *Re* = 1, *Pr* = 0.71and *J* = 1

VII. CONCLUSIONS

The major conclusions may be drawn from the present investigations are as follows:

This work is focused on the study of mixed magneto convection of fluid enclosed in a lid-driven cavity heated from wavy vertical surface. Effects of Prandtl number, Reynolds number and waviness are highlighted to explore their impacts on flow structure and heat transfer characteristics. The heat transfer and the flow characteristics inside the enclosure has strong influence on Re and little influence on Prandtl number and waviness. For lower values of Prandtl number thermal diffusivity dominates, this means that conduction dominates over convection. On the other hand for higher values of Prandtl number convection dominates over conduction. The local Nusselt number is highest and velocity of the fluid is lowest for the largest value of Pr (=10). This is because the fluid with the highest Prandt number is capable to carry more heat away from heat source through dominant convection. For higher values Reynolds number lower effect on the other hand lower values of Reynolds number streamlines are distributed in the whole cavity i.e. the viscous force becomes more effective.

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