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Instruction: Choice Of Optimum Parameters Of Technical Systems At The Design Stage

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Abstract*—The paper presents a guide: selection of optimal parameters of technical systems *at a design stage. We have presented mathematical model of technical system in the first section. The model is created as a vector problem of mathematical programming. In model criteria (characteristics) are formed in the conditions of definiteness (functional dependence of each characteristic and restrictions on parameters is known) and in the conditions of uncertainty (there is no sufficient information on functional dependence of each characteristic on parameters). We have presented in the second section how the specification (Basic data) for creation of model is formed. Basic data are formed by the designer of technical system (the designer, the client, the customer). In the third section performer (Mashunin Yu. K.) forms mathematical model of technical system. The performer solves a vector problem at equivalent criteria, and reports results to the customer. If necessary the performer solves a vector problem with a priority of this or that criterion and reports results to the customer.

Keywords—Modeling technical systems, Vector optimization, Optimum decision-making, the decision with a criterion priority

I. Introduction (*Mathematical model of technical systems in the conditions of definiteness and uncertainty in total*)

The Technique (instruction): "The choice of optimum parameters of technical systems at a design stage" is result of thirty summer researches of the author in the field of vector optimization [1 - 9]. *Conditions of definiteness* are characterized by that functional dependence of each characteristic and restrictions on parameters of technical system [2 - 8] is known*.* *Conditions of uncertainty* are characterized by that there is no sufficient information on functional dependence of each characteristic and restrictions from parameters [4 - 9].

In real life of a condition of definiteness and uncertainty are combined. The model of technical system also has to reflect these conditions. We will present model of technical system in the conditions of definiteness and uncertainty in total :

$$Opt\ F(X) = \{\mathbf{max}\ F_1(X) = \{\mathbf{max}\ f_k(X),\ k = \overline{1, K_1^{def}}\}, \quad (1)$$

$$\mathbf{max}\ I_1(X) \equiv \{\mathbf{max}\{f_k(X_i,\ i=\overline{1,M})\}^{\mathrm{T}},\ k=\overline{1, K_1^{unc}}\}, \quad (2)$$

$$\mathbf{min}\ F_2(X) = \{\mathbf{min}\ f_k(X),\ k = \overline{1, K_2^{def}}\}, \quad (3)$$

$$\mathbf{min}\ I_2(X) \equiv \{\mathbf{min}\{f_k(X_i,\ i=\overline{1,M})\}^{\mathrm{T}},\ k=\overline{1, K_2^{unc}}\}\}, \quad (4)$$

$$\text{at restrictions } x_j^{\min} \le x_j \le x_j^{\max},\ j = \overline{1,N}, \quad (5)$$

where X - a vector of operated variable (design data) equivalent (1); $F(X)=\{F_1(X)\ F_2(X)\ I_1(X),\ I_2(X)\}$ - vector criterion which everyone a component represents a vector of criteria (characteristics) of technical system (2) which functionally depend on discrete values of a vector of variables X ; $F_1(X)$, $F_2(X)$ - a set of the max and min functions respectively; $I_1(X)$ и $I_2(X)$ set of matrixes of max and min respectively; K_1^{def}, K_2^{def} (*definiteness*), K_1^{unc}, K_2^{unc} (*uncertainty*) the set of criteria of *max* and *min* created in the conditions of definiteness and uncertainty;

in (9) $f_k^{\min} \le f_k(X) \le f_k^{\max}$, $k=\overline{1,K}$ – a vector function of the restrictions imposed on functioning of technical system $x_j^{\min} \le x_j \le x_j^{\max}$, $j = \overline{1,N}$ – parametrical restrictions.

II. Terms of Reference: "The choice of the Optimal Technical Parameters of the System" (Performed Technical System Designer)

A. *Total*

We will consider a task "Numerical modeling of technical system" in which data on some set of functional characteristics (definiteness conditions), discrete values of characteristics (an uncertainty condition) and the restrictions imposed on functioning of technical system are known. The numerical problem of modeling of technical system is considered with equivalent criteria and given priority of criterion.

B. *The technical assignment*

It is given. The technical system, which functioning is defined by three parameters[1] $X=\{x_1, x_2, x_3\}$ – a vector

[1] Practical problems of simulation of technical systems on this algorithm can be solved with dimensionality of parameters X more than two N>2. The structure of the software becomes complicated. Geometrical interpretation of N=3,4 … isn't possible. The choice of two parameters

(operated) variables. Basic data for the solution of a task are fore characteristics (criterion) of $F(X)=\{f_1(X), f_2(X), f_3(X), f_4(X)\}$, which size of an assessment depends on a vector of X.

Definiteness conditions. For characteristics of $f_3(X)$, $f_4(X)$ functional dependence on parameters X (a definiteness condition) is known:

$$f_1(X) = 50+11.55*x_1+3.55*x_2+1.0*x_3+0.0144*x_1*x_2-0*x_1*x_3+0*x_2*x_3-0.07*x_1^2-0.07*x_2^2-0*x_3^2. \quad (6)$$

Parametrical restrictions:

$$25 \le x_1 \le 100,\ 25 \le x_2 \le 100,\ 25 \le x_3 \le 100. \quad (7)$$

1) Uncertainty condition. For the second, third and fourth characteristic results of experimental data are known: sizes of parameters and corresponding characteristics. Numerical values of parameters X and characteristics of $y_2(X)$, $y_3(X)$, $y_4(X)$ are presented in table 1.

TABLE I. NUMERICAL VALUES OF PARAMETERS AND CHARACTERISTICS OF TECHNICAL SYSTEM.

x1	*x2*	*x3*	*y2(X)→ max*	*y3(X) → min*	*y4(X) → min*
25	25	25	1197.2	90.25	25.25
25	25	50	1232.8	100.75	35.50
25	25	75	1393.3	86.25	48.125
25	25	100	1303.8	84.25	55.75
25	50	25	2232.3	210.25	20.75
25	50	50	2267.7	233.25	31.00
25	50	75	2303.2	206.25	41.125
25	50	100	2338.8	204.25	51.25
25	75	25	3077.2	410.25	16.25
25	75	50	2862.8	408.25	25.25
25	75	75	3148.3	443.75	36.625
25	75	100	3183.7	404.25	46.75
25	100	25	3732.3	690.25	11.75
25	100	50	3767.7	688.25	22.00
25	100	75	3803.2	686.25	32.125
25	100	100	3838.8	684.25	42.25
50	25	25	1245.3	94.25	55.00
50	25	50	1303.8	90.25	71.50
50	25	75	1374.7	86.25	91.75
50	25	100	1445.8	82.25	112.125
50	50	25	2267.7	239.25	42.25
50	50	50	2338.8	210.25	62.50
50	50	75	2409.7	206.25	82.75
50	50	100	2480.8	202.25	103.125
50	75	25	3112.8	414.25	33.25
50	75	50	3183.7	410.25	53.50
50	75	75	3379.8	406.25	86.25
50	75	100	3325.8	402.25	94.125
50	100	25	3767.7	744.25	24.25
50	100	50	3838.8	690.25	44.50
50	100	75	3909.7	686.25	64.75
50	100	100	3980.8	682.25	85.125
75	25	25	1268.3	102.25	90.625
75	25	50	1374.7	96.25	121.00
75	25	75	1481.3	90.25	151.50
75	25	100	1587.8	84.25	182.00
75	50	25	2303.2	222.25	77.125
75	50	50	2409.7	216.25	107.50
75	50	75	2516.2	210.25	138.00
75	50	100	2622.7	204.25	168.50
75	75	25	3148.3	422.25	63.625
75	75	50	3254.8	416.25	94.00
75	75	75	3361.3	410.25	124.50
75	75	100	3467.8	404.25	155.00
75	100	25	3803.2	702.25	50.125
75	100	50	3909.7	696.25	80.50
75	100	75	4016.3	690.25	111.00
75	100	100	4122.7	684.25	141.50
100	25	25	1303.8	114.25	143.50
100	25	50	1445.8	106.25	184.125
100	25	75	1587.8	98.25	224.75
100	25	100	1729.7	90.25	265.25
100	50	25	2338.8	234.25	125.50
100	50	50	2480.8	226.25	166.125
100	50	75	2622.7	218.25	206.75
100	50	100	2764.7	210.25	247.25
100	75	25	3183.7	434.25	107.50
100	75	50	3325.8	426.25	148.125
100	75	75	3467.8	418.25	188.75
100	75	100	3609.8	410.25	229.25
100	100	25	3838.8	714.25	89.50
100	100	50	3980.8	706.25	130.125
100	100	75	4122.7	698.25	233.25
100	100	100	4264.8	690.25	211.25

In the made decision, assessment size of the first and the third characteristic (criterion) is possible to receive above: $f_1(X)$→**max** $y_3(X)$→**max**; for second and fourth characteristic is possible below: $y_2(X)$→**min** $y_4(X)$→**min**. Parameters $X=\{x_1, x_2, x_3\}$ change in the following limits: $x_1, x_2, x_3 \in [25. 50. 75. 100.]$.

It is required. To construct model of technical system in the form of a vector problem. To solve a vector problem with equivalent criteria. To choose priority criterion. To establish numerical value of priority criterion. To make the best decision (optimum).

Note. The author developed the software for three parameters: $X=\{x_1, x_2, x_3\}$ and six characteristics of $F(X)=\{f_1(X), \dots, f_6(X)\}$. On each task the program is set up individually. In case of desire the author can increase the number of parameters to five: $X=\{x_1, \dots, x_5\}$. In model criteria with conditions of uncertainty can change from zero to six.

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III. SOLUTION OF A PROBLEM: "CHOICE OF OPTIMUM PARAMETERS OF TECHNICAL SYSTEM" (METHODOLOGY OF MODELING OF TECHNICAL SYSTEM IN THE CONDITIONS OF DEFINITENESS AND UNCERTAINTY)

Please take note of the following items when proofreading spelling and grammar:

selected from three (N '=2) ⊂ (N=3) is possible. In this direction it is carried further researches and development of the appropriate algorithms.

A. Creation of Mathematical Model of Technical System

1. Construction in the conditions of definiteness is defined by functional dependence of each characteristic and restrictions on parameters of technical system. In our example two characteristics (6) and restrictions (7) are known. Uniting them, we will receive a vector task with two criteria:

$$opt\ F(X)=\{max\ F_1(X)=\{max\ f_1(X)\equiv f_1(X)=50.0+11.55*x_1+3.55*x_2+1.0*x_3+0.0144*x_1*x_2-0.0*x_1*x_3+0.0*x_2*x_3-0.07*x_1^2-0.07*x_2^2-0.0*x_3^2\}\}. \quad (8)$$

Parametrical restrictions:

$$25\le x_1\le 100,\ 25\le x_2\le 100,\ 25\le x_3\le 100. \quad (9)$$

These data are used further at creation of mathematical model of technical system.

2. Construction in the conditions of uncertainty consists in use of the qualitative and quantitative descriptions of technical system received by the principle "entrance exit" in table 1. Transformation of information (basic data of $y_2(X)$, $y_3(X)$, $y_4(X)$) to a functional type of $f_2(X)$, $f_3(X)$, $f_4(X)$ is carried out by use of mathematical methods (the regression analysis). Basic data of table 1 are created in Matlab system in the form of a matrix

$$I=|X,Y|=\{x_{i1}\ x_{i2}\ x_{i3}\ y_{i2}\ y_{i3}\ y_{i4},\ i=\overline{1,M}\}. \quad (10)$$

For each set experimental these y_k, $k=\overline{2,4}$ function of regression on a method of the smallest squares in *Matlab* system is formed. A_k,- polynom defining interrelation of parameters of $X_i=\{x_{1i}, x_{2i}, x_{3i}\}$ (10) and functions $\bar{y}_{ki}=f(X_i,A_k)$, $k=\overline{2,4}$ is constructed. As a result of calculations we received system of coefficients of $A_k=\{A_{0k}, A_{1k}, \ldots, A_{9k}\}$ which define coefficients of a polynom (function):

$$f_k(X,A)=A_{0k}+A_{1k}x_1+A_{2k}x_1^2+A_{3k}x_2+A_{4k}x_2^2+A_{5k}x_3+A_{6k}x_3^2+A_{7k}x_1*x_2+A_{8k}x_1*x_3+A_{9k}x_2*x_3,\ k=\overline{2,4}. \quad (11)$$

As a result of calculations of coefficients of A_k, $k=2$, we received the $f_2(X)$ function:

$$f_2(X)=-53.875+0.7359*x_1+51.3703*x_2+0.3516*x_3+0.0072*x_1*x_2+0.0519*x_1*x_3+0.0005*x_2*x_3-0.0066*x_1^2-0.1454*x_2^2+0.0003*x_3^2. \quad (12)$$

As a result of calculations of coefficients of A_k, $k=3$, we received the $f_3(X)$ function:

$$f_3(X)=55.7188-0.1187*x_1+0.1844*x_2-0.0438*x_3-0.0002*x_1*x_2-0.0023*x_1*x_3-0.0011*x_2*x_3+0.0032*x_1^2+0.0634*x-0*x_3^2, \quad (13)$$

As a result of calculations of coefficients of A_k, $k=4$, we received the $f_4(X)$ function:

$$f_4(X)=25.6484-0.2967*x_1-0.3384*x_2+0.1433*x_3-0.0048*x_1*x_2+0.0169*x_1*x_3+0.0009*x_2*x_3+0.012*x_1^2+0.0014*x_2^2-0.0018*x_3^2. \quad (14)$$

Parametrical restrictions are similar (9).

3. Creation of mathematical model of technical system in the conditions of definiteness and uncertainty.

For creation of mathematical model of technical system we used: the functions received conditions of definiteness (8) and uncertainty (12), (13), (14); parametrical restrictions (9).

B. Decision-making on the basis of technical system model at equivalent criteria

1 Algorithm. The decision in problems of vector optimization with equivalent criteria

The solution of a vector problem (16)-(20) with equivalent criteria was submitted as sequence of steps.

- *Step 1*. Problems (16)-(20) were solved by each criterion separately, thus used the function *fmincon* (…) of *Matlab* system [16], the appeal to the function *fmincon* (…) is considered in [10]. As a result of calculation for each criterion we received optimum points: X_k^* and $f_k^*=f_k(X_k^*)$, $k=\overline{1,K}$ – sizes of criteria in this point, i.e. the best decision on each criterion:

$X_1^*=\{x_1=86.02,\ x_2=34.2,\ x_3=100\}$, $f_1^*=f_1(X_1^*)=-707.47$;

$X_2^*=\{x_1=25,\ x_2=25,\ x_3=25\}$, $f_2^*=f_2(X_2^*)=1200.0$;

$X_3^*=\{x_1=100,\ x_2=100,\ x_3=25\}$, $f_3^*=f_3(X_3^*)=-724.69$;

$X_4^*=\{x_1=25,\ x_2=100,\ x_3=25\}$, $f_4^*=f_4(X_4^*)=9.16$.

Restrictions (20) and points of an optimum in coordinates $\{x_1, x_2\}$ are presented on figure 1.

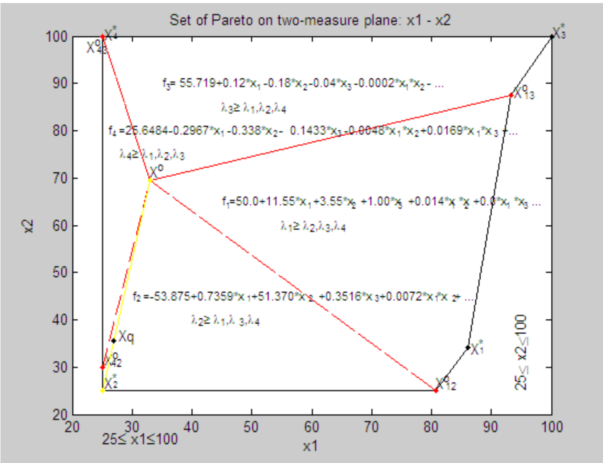

Figure 1. Pareto's great number, $S^o\subset S$ in two-dimensional system of coordinates

- Step 2. We defined the worst unchangeable part of each criterion (anti-optimum):

$X_1^0=\{x_1=25,\ x_2=100,\ x_3=25\}$, $f_1^0=f_1(X_1^0)=11.0$;

$X_2^0=\{x_1=100,\ x_2=100,\ x_3=100\}$, $f_2^0=f_2(X_2^0)=-4270.9$;

$X_3^0=\{x_1=43.5,\ x_2=20,\ x_3=80\}$, $f_3^0=f_3(X_3^0)=85.0$;

$X_4^0=\{x_1=100,\ \ x_2=25,\ \ x_3=100\}, f_4^0=f_2(X_4^0)=-263.97$. (Top index zero).

• Step 3. The system analysis of a set of points, optimum according to Pareto is made, (i.e. the analysis by each criterion). In points of an optimum of $X^*=\{X_1^*, X_2^*, X_3^*, X_4^*\}$ sizes of criterion functions of $F(X^*)=\|f_q(X_k^*)\|_{q=\overline{1,K}}^{k=\overline{1,K}}$ determined. Calculated a vector of $D=(d_1\ d_2\ d_3\ d_4)^T$ - deviations by each criterion on an admissible set of $\boldsymbol{S}$: $d_k=f_k^*-f_k^0$, $k=\overline{1,4}$, and matrix of relative estimates of

$$\lambda(X^*)=\|\lambda_q(X_k^*)\|_{q=\overline{1,K}}^{k=\overline{1,K}},\ \text{where } \lambda_k(X)=(f_k^*-f_k^0)/d_k.$$

$$F(X^*)=\begin{vmatrix} 707.5 & 2055.1 & 127.1 & 209.6 \\ 374.0 & 1200.0 & 96.1 & 28.7 \\ 329.0 & 3848.7 & 724.7 & 95.1 \\ 11.0 & 3704.1 & 701.9 & 9.2 \end{vmatrix},\ D=\begin{vmatrix} 696.5 \\ -3070.9 \\ 639.7 \\ -254.8 \end{vmatrix},$$

$$\lambda(X^*)=\begin{vmatrix} 1.0000 & 0.7216 & 0.0658 & 0.2132 \\ 0.5212 & 1.0000 & 0.0174 & 0.9232 \\ 0.4566 & 0.1375 & 1.0000 & 0.6628 \\ 0 & 0.1846 & 0.9644 & 1.0000 \end{vmatrix}$$

Discussion. The analysis of sizes of criteria in relative estimates showed that in points of an optimum of $X^*=\{X_1^*, X_2^*, X_3^*, X_4^*\}$ the relative assessment is equal to unit. Other criteria there is much less than unit. It is required to find such point (parameters) at which relative estimates are closest to unit. The step 4 is directed on the solution of this problem.

Step 4. Creation of λ-problem is carried out in two stages: originally the maximine problem of optimization with the normalized criteria is under construction:

$$\lambda^o=\max_x \min_k \lambda_k(X),\ G(X)\le 0,\ X\ge 0,$$

which at the second stage was transformed to a standard problem of mathematical programming (λ-problem):

$$\lambda^o=max\ \lambda, \qquad (21)$$

at restrictions

$$\lambda-\frac{50.0+11.55*x_1\ldots+0.014*x_1*x_2\ldots-0.07*x_1^2\ldots-f_1^o}{f_1^*-f_1^o}\le 0, \quad (22)$$

$$\lambda-\frac{55.71-0.118*x_1\ldots-0.002*x_1*x_2\ldots-0.0032*x_1^2\ldots-f_3^o}{f_3^*-f_3^o}\le 0, \quad (23)$$

$$\lambda-\frac{53.87+0.7359*x_1+\ldots.-0.0519*x_1*x_2\ldots+0.0066*x_1^2\ldots-f_2^o}{f_2^*-f_2^o}\le 0, \quad (24)$$

$$\lambda-\frac{25.6484-0.2967*x_1\ldots-0.0048*x_1*x_2\ldots+0.012*x_1^2\ldots-f_4^o}{f_4^*-f_4^o}\le 0, \quad (25)$$

$$0\le\lambda\le 1,\ \ 25\le x_1\le 100,\ 25\le x_2\le 100,\ 25\le x_3\le 100, \qquad (26)$$

where the vector of unknown had dimension of $N+1$: $\boldsymbol{X}=\{x_1, \ldots, x_N, \lambda\}$.

• Step 5. Solution of a λ-problem. For the solution of a λ-problem we use the function fmincon(…), [10]:

[Xo,Lo]=fmincon('Z_TehnSist_4Krit_L',X0,Ao,bo,Aeq,beq,lbo,ubo,'Z_TehnSist_LConst',options).

As a result of the solution of a vector problem of mathematical programming (16)-(20) at equivalent criteria and λ-problem corresponding to it (21)-(26) received: $\boldsymbol{X^o}=\{X^o, \lambda^o\}=\{X^o=\{x_1=33.027, x_2=69.54, x_3=25.0, \lambda^o=0.4459\}\}$ - an optimum point – design data of technical system, point $\boldsymbol{X^o}$ is presented in figure 1; $f_k(X^o)$, $k=\overline{1,K}$ - sizes of criteria (characteristics of technical system): $\{f_1(X^o)=321.5, f_2(X^o)=2901.7, f_3(X^o)=370.2, f_4(X^o)=19.1\}$; (27)

$\lambda_k(X^o)$, $k=\overline{1,K}$ - sizes of relative estimates: $\{\lambda_1(X^o)=0.4459,\ \lambda_2(X^o)=0.4459,\ \lambda_3(X^o)=0.4459,\ \lambda_4(X^o)=0.9609\}$; (28)

$\lambda^o=0.4459$ is the maximum lower level among all relative estimates measured in relative units: : $\lambda^o=min\,(\lambda_1(X^o), \lambda_2(X^o), \lambda_3(X^o), \lambda_4(X^o))=0.4459$. A relative assessment - λ^o call the guaranteed result in relative units, i.e. $\lambda_k(X^o)$ and according to the characteristic of technical $f_k(X^o)$ system it is impossible to improve, without worsening thus other characteristics.

Discussion. We will notice that according to the theorem 2 [5, 234 p.], in $\boldsymbol{X^o}$ point criteria 1, 2, 3 are contradictory. This contradiction is defined by equality of $\lambda_1(X^o)=\lambda_2(X^o)=\lambda_3(X^o)=\lambda^o=0.4459$, and other criteria an inequality of $\{\lambda_4(X^o)=0.9609\}>\lambda^o$.

Thus, the theorem 2 [5, 234 p.] forms a basis for determination of correctness of the solution of a vector task. In a vector problem of mathematical programming, as a rule, for two criteria equality is carried out: $\lambda^o=\lambda_q(X^o)=\lambda_p(X^o)$, $q, p\in\boldsymbol{K}$, $X\in S$, (in our example of such criteria three) and for other criteria is defined as an inequality: $\lambda^o\le\lambda_k(X^o)\ \ \forall k\in\boldsymbol{K}$, $q\ne p\ne k$.

• Step 6. Creation of geometrical interpretation of results of the decision in three to measured system of coordinates

In an admissible set of points of $\boldsymbol{S}$ formed by restrictions (26), optimum points $X_1^*, X_2^*, X_3^*, X_4^*$ united in a contour, presented a set of points, optimum across Pareto, to $\boldsymbol{S^o}\subset\boldsymbol{S}$. For specification of border of a great number of Pareto calculated additional points: $X_{12}^o, X_{13}^o, X_{42}^o, X_{34}^o$ which lie between the corresponding criteria. For definition of a point of X_{12}^o the vector problem was solved with two criteria (21), (22), (23), (26). Results of the decision:

$X_{12}^o=\{80.78\ \ \ 25.0\ \ \ 55.89\}$, $\lambda^o(X_{12}^o)=0.9264$;

$F_{12}=\{656.2\ \ \ 1426.0\ \ \ 101.7\ \ \ 142.7\}$;

$L_{12}=\{\mathbf{0.9264}\ \ \ \mathbf{0.9264}\ \ \ 0.0261\ \ \ 0.4761\}$.

Other points X_{13}^o, X_{42}^o, X_{43}^o were similarly defined:

X^o_{13}={93.29 87.49 100.0}, $\lambda^o(X^o_{13})$=0. 7173;

F_{13} = {510.6 3924.4 543.8 206.2};

L_{13} = (**0.7173** 0.1128 **0.7173** 0.2267};

X^o_{42}={25.0 29.92 25.0}, $\lambda^o(X^o_{42})$=0. 9301;

F_{42} = {374.3 1414.5 114.0 27.0};

L_{42} = {0.5217 **0.9301** 0.0454 **0.9301**}

X^o_{43}={25.0 100.0 56.02}, $\lambda^o(X^o_{43})$=0. 8366;

F_{43} = {42.0 3757.6 695.4 25.0};

L_{43} = {0.0445 0.1672 **0.9541** **0.9541**};

Points: X^o_{12}, X^o_{13}, X^o_{42}, X^o_{43} are presented in figure 1. Coordinates of these points, and also characteristics of technical system in relative units of $\lambda_1(X)$, $\lambda_2(X)$, $\lambda_3(X)$, $\lambda_4(X)$ are shown in figure 2 in three measured space {x_1, x_2, λ}, where the third axis of λ - a relative assessment.

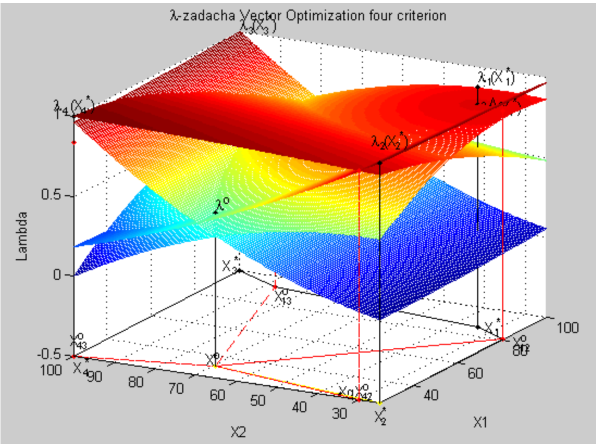

Figure 2. The solution of λ-problem in three-dimensional system of coordinates of x_1, x_2 and λ

C. Decision-making on the basis of technical system model at the set priority of criteria

2 Algorithm. The decision in problems of vector optimization with a criterion priority

The solution of a vector problem (16)-(20) with a criterion priority was submitted as sequence of steps.

• Step 1. *We solve a vector problem with equivalent criteria*. The algorithm of the decision is presented in section 3.2. Numerical results of the solution of a vector task are given above. Pareto's great number of $S^o \subset S$ lies between optimum points $X^*_1 X^o_{13} X^*_3 X^o_{43}\ X^*_4 X^o_{42}\ X^*_2 X^o_{12} X^*_1$.

We will carry out the analysis of a great number of Pareto $S^o \subset S$. For this purpose we will connect auxiliary points: X^o_{12}, X^o_{13}, X^o_{43}, X^o_{42}, with a point X^o which conditionally represents the center of a great number of Pareto. As a result have received four subsets of points $X \in \boldsymbol{S}^o_q \subset \boldsymbol{S}^o \subset \boldsymbol{S}$, $q=\overline{1,4}$. The subset of $\boldsymbol{S}^o_1 \subset \boldsymbol{S}^o \subset \boldsymbol{S}$ is characterized by the fact that the relative assessment of $\lambda_1 \geq \lambda_2, \lambda_3, \lambda_4$, i.e. in the field of $\boldsymbol{S}$ first criterion has a priority over the others. Similar to $\boldsymbol{S}^o_2$, $\boldsymbol{S}^o_3$, $\boldsymbol{S}^o_4$,- subsets of points where the second - the fourth criterion has a priority over the others respectively. Set of points, optimum across Pareto we will designate $\boldsymbol{S}^o = \boldsymbol{S}^o_1 \cup \boldsymbol{S}^o_2 \cup \boldsymbol{S}^o_3 \cup \boldsymbol{S}^o_4$. Coordinates of all received points and relative estimates are presented in two-dimensional space in fig. 1. These coordinates are shown in three measured space {x_1, x_2, λ} from a point X^*_4 in fig. 2 where the third axis of λ- a relative assessment. Restrictions of a set of points, optimum across Pareto, in fig. 2 it is lowered to -0.5 (that restrictions were visible). This information is also a basis for further research of structure of a great number of Pareto. *The person making decisions, as a rule, is the designer of technical system*. If results of the solution of a vector task with equivalent criteria don't satisfy the person making the decision, then the choice of the optimal solution is carried out from any subset of points of $\boldsymbol{S}^o_1$, $\boldsymbol{S}^o_2$, $\boldsymbol{S}^o_3$, $\boldsymbol{S}^o_4$..

• Step 2. *Choice of priority criterion of* $q \in \boldsymbol{K}$. From the theory (see the theorem 2 [10]) it is known that in an optimum point of X^o always there are two most inconsistent criteria, $q \in \boldsymbol{K}$ and $v \in \boldsymbol{K}$ for which in relative units exact equality is carried out: $\lambda^o = \lambda_q(X^o) = \lambda_p(X^o)$, q, $v \in \boldsymbol{K}$, $X \in \boldsymbol{S}$, and for the others it is carried out inequalities: $\lambda^o \leq \lambda_k(X^o)\ \ \forall k \in \boldsymbol{K}$, $q \neq v \neq k$.

In model of technical system (16)-(20) and the corresponding λ-problem (21)-(26) such criteria are the first, second and third:

$$\lambda^o = \lambda_1(X^o) = \lambda_2(X^o) = \lambda_3(X^o) = 0.4459. \qquad (29)$$

We will show them in figure 3.

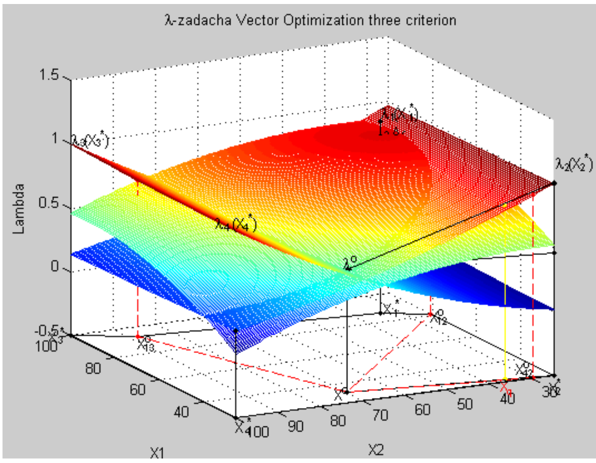

Figure 3. The solution of λ-problem (1, 2, 3 criterion) in three-dimensional system of coordinates of x_1, x_2 and λ

As a rule, the criterion which the decision-maker would like to improve gets out of couple of contradictory criteria. Such criterion is called "priority criterion", we will designate it $q=2\in\boldsymbol{K}$. This criterion is investigated in interaction with the first criterion of $k=1\in\boldsymbol{K}$.

On the display the message is given:

q=input ('Enter priority criterion (number) of q =') - Have entered: q=2.

• Step 3. *Numerical limits of change of size of a priority of criterion of* $q=2\in\boldsymbol{K}$ are defined.

For priority criterion of $q=2$ numerical limits in physical units upon transition from a point of an optimum of X^o to the point of X^*_q received on the first step are defined. Information about the criteria for q=2 are given on the screen:

$f_q(X^o)$ =2901.68 $\leq f_q(X)\leq$ 1200.0=$f_q(X^*_q)$, $q\in\boldsymbol{K}$. (30)

In relative units the criterion of q=2 changes in the following limits:

$\lambda_q(X^o)$ =0.4459$\leq$ $\lambda_q(X)\leq$1= $\lambda_q(X^*_q)$, $q=2\in\boldsymbol{K}$.

These data it is analyzed.

• Step 4. Choice of size of priority criterion. q∈K. (Decision-making). The message is displayed: "Enter the size of priority criterion fq=" – we enter, for example, fq =1600.

• Step 5. *Calculation of a relative assessment*.

For the chosen size of priority criterion of f_q =1600 the relative assessment is calculated:

$$\lambda_q=\frac{f_q - f_q^o}{f_q^* - f_q^o}=\frac{1600 - 4279.9}{1200.0 - 4279.9}=0.8697, \quad (31)$$

which upon transition from X^o point to X^*_q according to (28) lies in limits:

0. 4459 =$\lambda_2(X^o)\leq\lambda_2$=0.8697$\leq\lambda_2(X^*_2)$=1, $q\in\boldsymbol{K}$..

• ***Step*** 6. *Calculation of coefficient of linear approximation*.

Assuming linear nature of change of criterion of $f_q(X)$ in (30) and according to a relative assessment of $\lambda_q(X)$, using standard methods of linear approximation, we will calculate proportionality coefficient between $\lambda_q(X^o)$, λ_q, which we will call ρ:

$$\rho=\frac{\lambda_q-\lambda_q(X^o)}{\lambda_q(X^*_q)-\lambda_q(X^o)}=\frac{0.8697 - 0.4459}{1 - 0.4459}=0.\ 7649,\ q=2. \quad (32)$$

• ***Step*** 7. *Calculation of coordinates of priority criterion with the size* f_q.

Assuming linear nature of change of a vector of $X^q=\{x_1\ x_2\}$, q=2 we will determine coordinates of a point of priority criterion with the size f_q =1600 with a relative assessment (31):

$$X^q=\{x_1=X^o(1)+\rho(X^*_q(1)-X^o(1))$$

$$x_2=X^o(2)+\rho(X^*_q(2)-X^o(2))\}.$$

where $X^o=\{x_1=33.02,\ x_2=69.54\}$, $X^*_2=\{x_1=25,\ x_2=25\}$.

As a result of calculations we have received point coordinates:

$X^q=\{x_1=26.88,\ x_2=69.54\}$. (33)

Step 8. *Calculation of the main indicators of a point of* X^q.

For the received X_q point, we will calculate:

all criteria in physical units $f_k(X^q)=\{f_k(X^q),\ k=\overline{1,K}\}$:

$f(X^q)=\{f_1(x^q)$=386.5, $f_2(x^q)$=1651.5, $f_3(x^q)$=137.9, $f_4(x^q)$=26.1};

all relative estimates of criteria $\lambda^q=\{\lambda^q_k,\ k=\overline{1,K}\}$, $\lambda_k(X^q)=\frac{f_k(X^q)-f_k^o}{f_k^*-f_k^o}$, $k=\overline{1,K}$:

$\lambda_k(x^q)=\{\lambda_1(x^q)$=0.5392, $\lambda_2(x^q)$=0.853, $\lambda_3(x^q)$= 0.0827, $\lambda_4(x^q)$=0.9334};

vector of priorities $P^q=\{p^q_k=\frac{\lambda_q(X^q)}{\lambda_k(X^q)},\ k=\overline{1,K}\}$:

$P^q=[p^2_1$ =1.5820, p^2_2 =1.0, p^2_3 =10.3123, p^2_4 =0.9139];

minimum relative assessment: minLXq=min(LXq): minLXq=min($\lambda_k(X^q)$) = 0.0827;

relative assessment taking into account a criterion priority:

λ^{oo}=min ($p^2_1\lambda_1(X^q)$ =0.7564, $p^2_2\lambda_2(X^q)$ =0.7564, $p^2_3\lambda_3(X^q)$ =0.7564 , $p^2_4\lambda_4(X^q)$) =0.7564).

Any point from Pareto's set $\boldsymbol{X}^o_t=\{\lambda^o_t,\ X^o_t\}\in\boldsymbol{S}^o$ *can be similarly calculated*.

Analysis of results. The calculated size of criterion $f_q(X^o_t)$, $q\in\boldsymbol{K}$ is usually not equal to the set f_q. The error of the choice of $\Delta f_q=|f_q(X^o_t) - f_q|$ =|1651.5-1600|=51.5 is defined by an error of linear approximation, $\Delta f_{q\%}$= 3.2%.

In the course of modeling parametrical restrictions (20) can be changed, i.e. some set of optimum decisions is received. Choose a final version which in our example included from this set of optimum decisions:

• parameters of technical system $X^o=\{x_1=33.03,\ x_2=69.54,\ x_3=25.0\}$;

• the parameters of the technical system at a given priority criterion q=2: $X^q=\{x_1=26.88,\ x_2=35.47,\ x_3=25.0\}$.

IV. GEOMETRICAL INTERPRETATION OF RESULTS OF THE DECISION IN THREE TO MEASURED SYSTEM OF COORDINATES IN PHYSICAL UNITS

We represent these parameters in a two-dimensional x_1, x_2 and three dimensional coordinate system x_1, x_2 and λ in Fig.1, 2, 3, and also in physical units for each function $f_1(X)$, … , $f_4(X)$ on Fig. 4, ... , 7, respectively.

The first characteristic $f_1(X)$ in physical units show in Fig. 4.

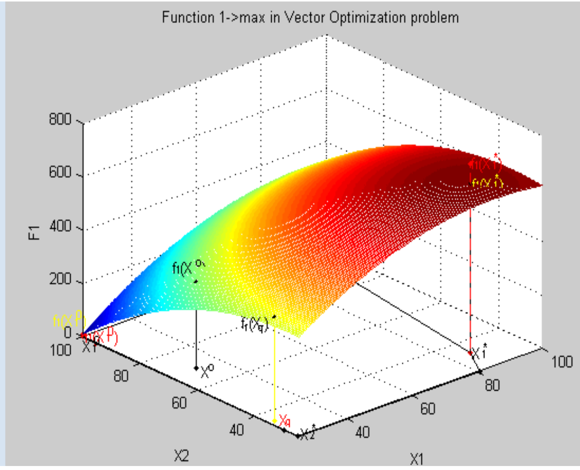

Figure 4. The first characteristics of $f_1(X)$ of technical system in natural indicator

In point X^o, X^q of the second characteristic of $f_2(X)$ will assume to the look presented in figure 5.

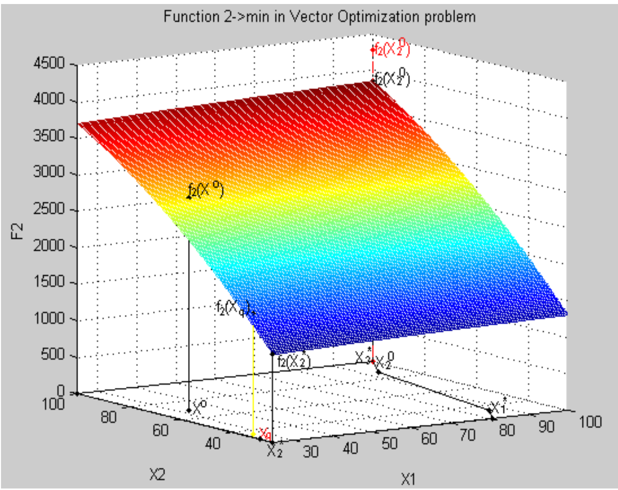

Figure 5. The second characteristics of $f_2(X)$ of technical system in natural indicator

In point X^o, X^q of the third characteristic of $f_3(X)$ will assume to the look presented in figure 6;

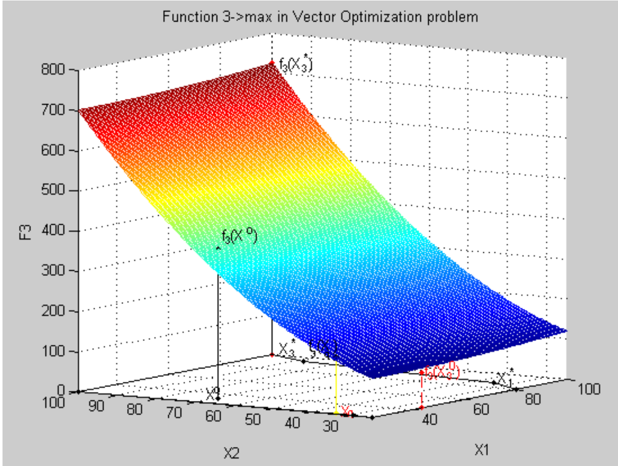

Figure 6. The third characteristics of $f_3(X)$ of technical system in natural indicator

In point X^o, X^q of the fourth characteristic of $f_4(X)$ will assume to the look presented in figure 7;

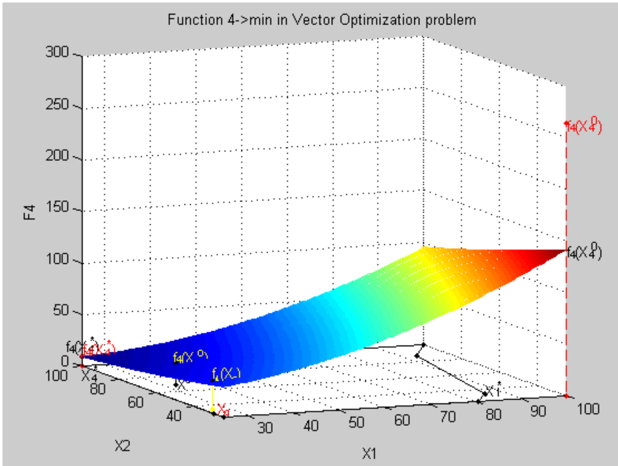

Figure 7. The fourth characteristics of $f_4(X)$ of technical system in natural indicator

Collectively, *the submitted version*:

- point - X^o;
- characteristics of $f_1(X^o)$, $f_2(X^o)$, $f_3(X^o)$, $f_4(X^o)$;
- relative estimates of $\lambda_1(X^o)$, $\lambda_2(X^o)$, $\lambda_3(X^o)$, $\lambda_4(X^o)$;
- maximum λ^o relative level such that $\lambda^o \le \lambda_k(X^o)$ $\forall k \in \boldsymbol{K}$

- there is *an optimal solution with equivalent criteria* (characteristics), and the procedure for obtaining an acceptance of the optimal solution with equivalent criteria (characteristics).

- point – X^q;
- characteristics of $f_1(X^q)$, $f_2(X^q)$, $f_3(X^q)$, $f_4(X^q)$;
- relative estimates of $\lambda_1(X^q)$, $\lambda_2(X^q)$, $\lambda_3(X^q)$, $\lambda_4(X^q)$;
- maximum λ^o relative level such that $\lambda^o \le \lambda_k(X^q)$ $\forall k \in \boldsymbol{K}$

- there is an optimal solution at the set priority of the second criterion (characteristic) in relation to other criteria. Procedure of receiving a point is X^q *adoption of the optimal solution at the set priority of the second criterion*.

Theory of vector optimization, methods of solution of the vector problems with equivalent criteria and given priority of criterion can choose any point from the set of points, optimum across Pareto, and show the optimality of this point.

Conclusions

The problem of adoption of the optimum decision in difficult technical system on some set of functional characteristics is one of the most important tasks of the system analysis and design. In work the new technology (methodology) of creation of mathematical model of technical system in the conditions of definiteness and uncertainty in the form of a vector problem of mathematical programming is presented. For the first time in domestic and foreign literature, we have submitted the theory of vector optimization and methods for the choice of any point, from Pareto's great number. The principles of an optimality of a point are shown in the theory, first, at equivalent criteria, secondly, at the set criterion priority. These methods can be used at design of technical systems of various branches: electro technical [2], aerospace, metallurgical, etc. At creation of characteristics in the conditions of uncertainty regression methods of transformation of information are used. The methodology of modeling and adoption of the optimum decision is based on normalization of criteria and the principle of the guaranteed result (maxmin). Methods allow solving vector problems at equivalent criteria and with the set criterion priority. Results of the decision are a basis for decision-making on the studied technical system on all set of point's optimum across Pareto. This methodology has system character and can be used when modeling both technical and economic systems. Authors are ready to participate in the solution of vector problems of linear and nonlinear programming.

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[2] We mention the work of V.L. Levitskii "Simulation and Optimization of Parameters of Magnetoelectric Linear Inductor Electric Direct Current Motor" [7, p. 50–120]. It deals with designing an augmented electric motor (AEM) with its model reduced to vector mathematical programming problem (1)–(5). The vector of design parameters $X = (X_1, \ldots, X_5)$ consisted of X_1 for the air clearance δ, X_2 for the tooth pitch, X_3 for the number of teeth, X_4 for the height of the concentrator, and X_5 for the pole overlap coefficient. The vector of design criteria $F(X) = (f(X), p(X), \eta(X), \ldots)$ included $f(X)$ for the nominal towing force, $p(X)$ for the nominal power, $\eta(X)$ for the nominal efficiency and so on, ten indices in total. The central orthogonal plan of the second order was used to construct the dependencies of f on the listed design parameters X [7, p. 96]. The work "…Multiobjec_ tive Optimization of Static Modes of Mass_Exchange Processes by the Example of Absorption in Gas Separation" [15] is an example from another industry. Thus, experimental data both from the AEM problem and from similar ES of other industries can be represented as theoretical (system) problem (1)-(5).