Measurement Of The Growth In The Magnetic Field Of Pulsar By Using Spin Frequency

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***Abstract*—We use a method of measuring the growth or decay in the magnetic field for some pulsar according to a new formula used by Lin-Sen Li; this formula used pulse period P and its first and second derivative $\dot{P}$, $\ddot{P}$ or spin frequency υ and its first and second derivative $\dot{\upsilon}$, $\ddot{\upsilon}$. our calculation made on crab pulsar that show growth with time in its magnetic field and expand our calculation for two young pulsars where some of its case show growth and others decay with time depending on the value of the inclination angle of the magnetic dipole of pulsar.**

Keywords—pulsar, spin frequency, magnetic field, growth or decay .

1. Introduction

Usually the magnetic field of pulsars decay with time. Such is the case for a lot of pulsars, but the magnetic field for a distinct pulsar is growing. The young pulsar is possibly such a case when it is born. Afterward its magnetic field is decaying successively. When it arrive at old pulsar, its magnetic field is not decaying or nearly without variation or become a weak field. The magnetic field of Crab pulsar should be growing due to young pulsar (Lin-Sen Li 2009). The spin frequency (ν) of all pulsars is observed to decrease and the rate of this spin down ($\dot{\upsilon}$) is commonly related to the magnitude *B* of a dipolar magnetic field

rotating with the star. The spin-down of pulsars can be characterized by the braking index n, which is defined via $\dot{\nu} \propto \nu^n$ and, provided we are able to detect the long-term second time-derivative of the spin frequency $\ddot{\upsilon}$, it can be measured by (Cristóbal M 2012):

$$n = \frac{\upsilon\ddot{\upsilon}}{\dot{\upsilon}^2} \qquad (1)$$

Where *v* is the spin frequency, and $\dot{\nu}$ and $\ddot{\nu}$ are the first and the second derivativesOne of the famous pulsar is the Crab pulsar (PSR B0531+21) which was discovered in 1968 and since then it has been monitored closely. It is a fast rotating normal pulsar with period *P* ~ 33 ms, and a very high period derivative $\dot{P}$ ~ 4.2 × 10^{-13}, which implies a strong magnetic field on the crab pulsar $B_s \sim 4 \times 10^{12}$ G. If we describe the pulsar rotation by $\dot{\nu} = -K\upsilon^n$, with K a positive constant related to the magnetic field strength, then the braking index n equals 3 for a dipole magnetic field. The characteristic age of the pulsar, given by $\tau = P/[(n-1)\dot{P}] = P/(\dot{P})$, where $P = 1/v$ is the pulsar period, is equal to the actual age if the pulsar was born with a period much less than its present value. With the given period and period derivative, the Crab pulsar has a characteristic age of 1257 yr, quite close to its true age based on the Chinese record of a 'guest star' in 1054, which was in fact the supernova explosion where the pulsar was born and the Crab Nebula was formed (Na Wang et.al 2001).

2. Is the magnetic field for pulsar growth or decay?

To research this problem about the magnetic dipole field either growth or decay around the pulsar magnetosphere we adopted the formula exponential term:

$$B_p^2 = B_i^2\ i \exp(\pm\zeta t) \qquad (2)$$

Where B_p is the magnetic field at magnetic pole of pulsar and B_i is the field strength at $t = 0$. ξ is the coefficient of the magnetic decay or growth. If ξ is positive, the magnetic field of pulsar is growing; if it is negative, the magnetic field is decaying (Lin-Sen Li 2009).

We adapted the magnetic dipole field that depends on the inclination angle α with to the axis of rotation

2.1 The case of the inclination without variation

The equation of magnetic dipole radiation for pulsar can be written as (Shapiro & Teukolsky 1983) assuming that the magnetic inclination angle α = constant:

$$\dot{\Omega} = -\frac{B_p^2 \quad R^2 \quad \Omega^3 \quad \sin^2 \quad \alpha}{6C^3 \quad I} \qquad (3)$$

Where R, I and Ω denote radius, moment inertia and angular velocity of pulsar respectively.

Let Ω = 2π/P (P: pulse period) ; Inserting it into the equation (3) , we get

$$B_p^2 = \frac{3c^3}{2\pi^2}\frac{I}{R^6}\frac{1}{\sin^2 \alpha} P\dot{P} \quad (4)$$

Differentiating equation (4) with respect to time:

$$\frac{dB_p^2}{dt} = \frac{3c^3 I}{2\pi^2 R^6 \sin^2\alpha}\left(\dot{P}^2 + P\ddot{P}\right) \quad (5)$$

$$\frac{1}{B_p^2}\frac{dB_p^2}{dt} = \frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} \quad (6)$$

Let υ be the spin frequency of pulsar, Where *P=1/* υ and $\dot{P} = dp/dt$ and $\ddot{P} = dP^2/dt^2$ then easily we can write equation (6) in term of spin frequency:

$$\frac{1}{B_p^2}\frac{dB_p^2}{dt} = \frac{\ddot{\upsilon}}{\dot{\upsilon}} - 3\frac{\dot{\upsilon}}{\upsilon} \quad (7)$$

We substitute the formula (2) into the formulas (6) and (7), we obtain the final formula driven by (Lin-Sen Li 2009):

$$\pm\zeta = \frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} = \frac{\ddot{\upsilon}}{\dot{\upsilon}} - 3\frac{\dot{\upsilon}}{\upsilon} \quad (8)$$

The formula (8) is a formula for deciding that the magnetic field of pulsar is growth or decay . If ξ is positive, the magnetic field of pulsar is growing; if it is negative, the magnetic field is decaying (Lin-Sen Li 2009).

2.2 The case of the inclination with angle variation

If the pulsar inclination angle is vary with time ; we must adopted it in our

calculation, so with differentiating the formula (4) with respect to time, we get:

$$\frac{dB_p^2}{dt} = \frac{3c^3 I}{2\pi^2 R^6 \sin^2\alpha}\left(\dot{P}^2 + P\ddot{P}\right) - \frac{3c^3}{\pi^2 R^6}\left(IP\dot{P}\right)\cot\alpha\csc^2\alpha\frac{d\alpha}{dt}$$
(9)

The final expression for the magnetic field driven by (Lin-Sen Li 2009) will be:

$$\frac{1}{B_p^2}\frac{dB_p^2}{dt} = \frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} + 2\cot^2\alpha\left(\frac{\dot{P}}{P}\right) = \left(1 + 2\cot^2\alpha\right)\frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}}$$
(10)

In spin frequency; Equation (10) will be as follow:

$$\frac{1}{B_p^2}\frac{dB_p^2}{dt} = \frac{\ddot{\upsilon}}{\dot{\upsilon}} - \left(3 + 2\cot^2\alpha\right)\frac{\dot{\upsilon}}{\upsilon} \quad (11)$$

Substituting the formula (2) into the left side for the formulas (11) and (10), we get:

$$\pm\xi = \left(1 + 2\cot^2\alpha\right)\frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} = \frac{\ddot{\upsilon}}{\dot{\upsilon}} - \left(3 + 2\cot^2\alpha\right)\frac{\dot{\upsilon}}{\upsilon}$$
(12)

The formulas (8) and (12) are the formulas for judging the growth or decay of the magnetic field of pulsar.

3.Result and Discussion

Our result will applied to the crab pulsar using the same data presented in the paper of (Lin-Sen Li 2009) and expand to young pulsars named PSR J1846-0258 and PSR J1357-6429 with help of formulas (8) and (12) to test the growth or decay in the magnetic field for each pulsar.by using equation (8) the results are shown in the table No. (1) that the ±ζ factor will have positive value for the three set of data for the crab pulsar and for pulsar PSR J1846-0258 that means the magnetic field will be growth with the time so that ζ should be taken as positive while for the pulsar PSR J1357-6429 the ±ζ factor will be negative value which means the magnetic field is continuous decay with time and ζ should be taken as negative.

If the inclination angle is take into account, we use formula (12) for in our calculation; for crab pulsar the ζ factor will have positive value when angle lies between 0° ≤ α ≤ 180° so the magnetic field will be growth corresponding to this result depends on equation (2) as demonstrate in figure (1); but this result will not agree with the result present in the calculations of (Lin-Sen Li 2009) his result found that ζ factor will have positive value with 0° ≤ α ≤ 90° and negative value when 90° ≤ α ≤ 180° and this difference because he made his calculation on the cot (α) not on the $\cot^2(\alpha)$ as present in the formula (12). Our result show positive value for the magnetic field which means that the filed will be growth with the time for the crab pulsar.

Table No. (1) Measured and derived parameters for a given pulsars

Pulsar name	υ Hz	$\dot{\upsilon}$ 10^{-10} Hz Sec^{-1}	$\ddot{\upsilon}$ 10^{-21} Hz sec^{-2}	ζ 10^{-10}	Refs
Crab Pulsar Data 1	30.2137051	−3.8594	10.1	+0.098191804	3
Crab pulsar Data 2	29.836059670	−3.743460(3)	10.17(2)	+0.063323602	1
Crab pulsar Data 3	30.225	−3.86	10.240	+0.061883027	3
PSR J1846-0258	3.0621185502(4)	-.06664350(2)	2.725(3)	+0.2440235196407	6
PSR J1357-6429	6.020167772600	-0.1305395	1.1600	-0.823568825586560	5

Another application for formula (12) applied for two young pulsars, first one will be PSR J1846-0258 as shows in figure (2) where the ζ factor will have positive value for inclination angle lies between 0° to 180° so we can see that the magnetic field also growth with time upon to equation (2), while for pulsar named PSR J1357-6429 the ζ factor will have negative value for inclination angle lies between 15° to 165° and otherwise will be positive value so the magnetic field will decay for the inclination angle lies between 15° to 165° and show growth for the others angles. The last pulsar refer that the inclination angle of the pulsar effect on the growth or decay in the magnetic field while the other pulsar dose not effect with varying in the inclination angle.

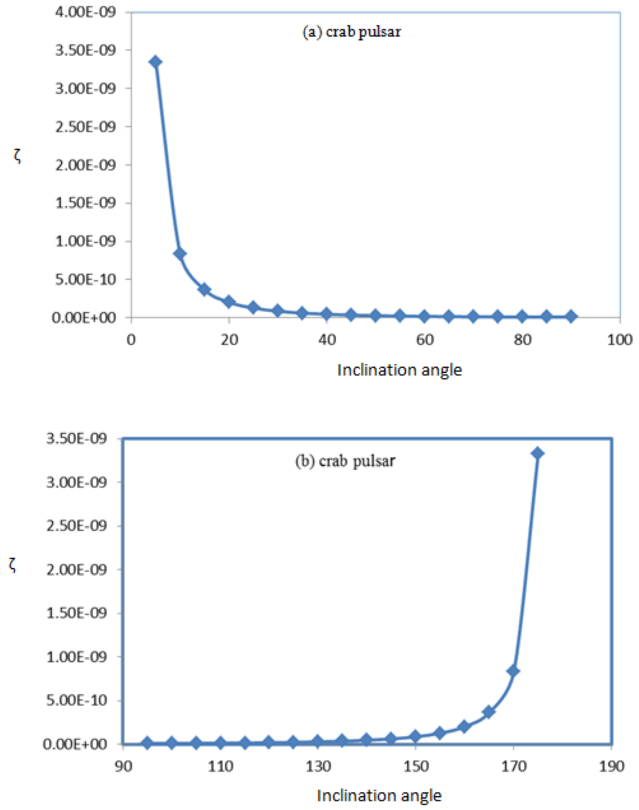

Figure (1) The measurement of the ζ factor with inclination angle of the magnetic field of the crab pulsar

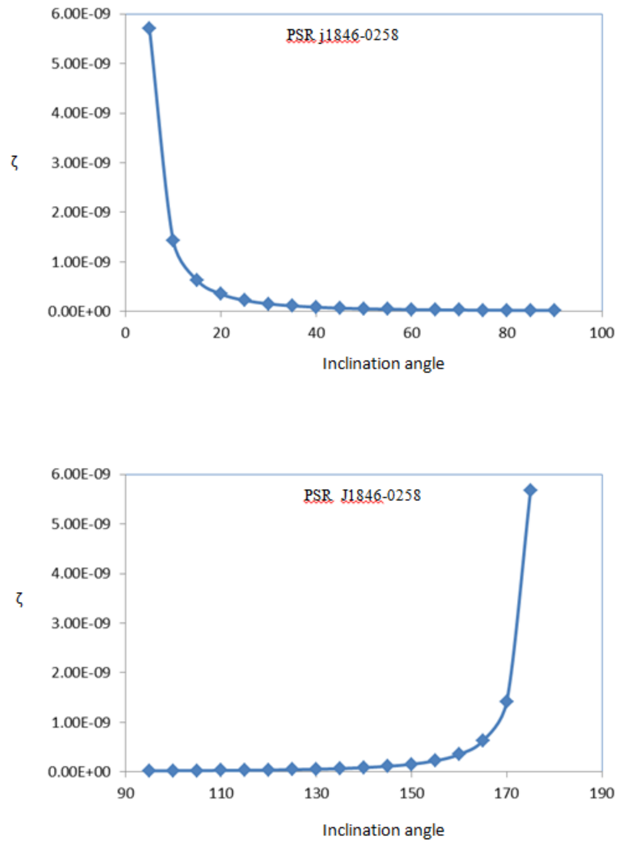

Figure (2) The measurement of the ζ factor with inclination angle of the magnetic field of the PSR J1846-0258

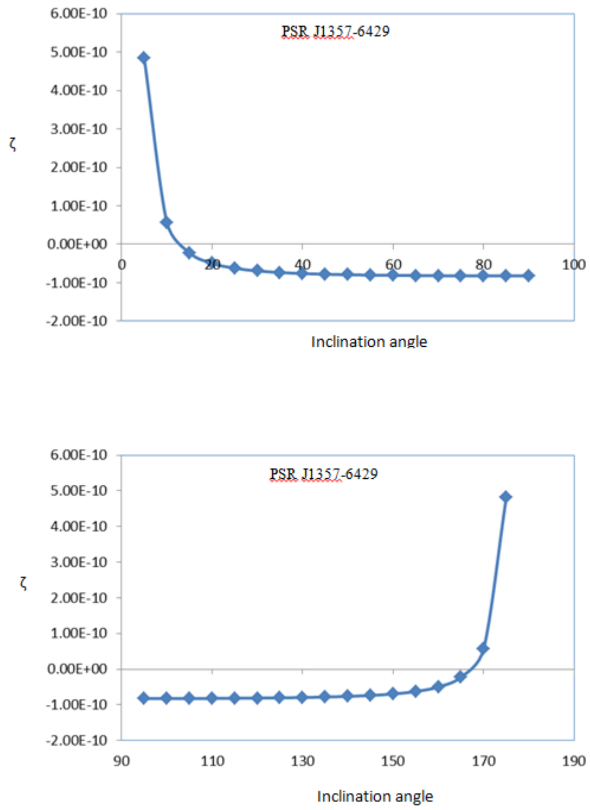

Figure (3) The measurement of the ζ factor with inclination angle of the magnetic field of the PSR J1357-6429

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