Effects Of Inertia And Gas Torques On Engine Crankshaft Systems

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***Abstract*—The effects of inertia and gas torques on the crankshaft system under load conditions were determined. This was achieved by running a Perkins 4 cylinder inline 4 stroke engine at constant speed with varying imep (4-10 bar) to obtain vibration amplitudes ranging from 0.003-0.006, 0.0025-0.0205, 0.0004-0.00047 and 0.018-0.025 for the 1st, 2nd, 3rd and 4th harmonic orders respectively. The same engine was run at constant imep with the speed varied (150-250 rads^{-1}) to obtain vibration amplitudes ranging from 0.0045-0.0049, 0.0055-0.056, 0.0015-0.001 and 0.02-0.0014 for the 1st, 2nd, 3rd and 4th harmonic orders respectively. The condition of the engine can be determined from the vibration amplitude. Thus, early detection of incipient faults is made possible. This will increase the reliability and hence availability of the engine.**

Keywords—inertia torque, gas torque, crankshaft, vibration amplitude.

INTRODUCTION

Internal combustion engines (ICE) are often used in various modes of transportation as well as in industrial and domestic applications. These modes of transportation include air, water and land. In these applications, the ICE can be used either for propulsion or power generation. In either application high reliability and availability are two of the most stringent demands made on the engine (Pounder*et al*, 1991) according to Ogbonnaya and Komako (2006). High reliability and availability can be achieved through condition monitoring as a component of preventive maintenance (Meier *et al*, 2009).

One of the components of an ICE which can be subjected to condition monitoring is the crankshaft system (Ogbonnaya, 2006). This paper focuses on the vibration amplitudes of the crankshaft system as a source of condition monitoring programme for preventive maintenance.

The specific objectives of this study are to:

i. Determine the effects of inertia and gas torques on the crankshaft system under load conditions.

i. Monitor the changes in vibration amplitude of the system under normal operating conditions in order to enable early detection faults.

iii. Improve engines reliability and availability through performing remedial action(s) to correct incipient faults once detected.

iv. Prevent downtime.

1. Crankshaft System and Analysis

A crankshaft system of an ICE can be divided into sections comprising the reciprocating masses of each cylinder, the pulley and the flywheel. These are rigidly connected together by shaft sections. A schematic illustration of the crankshaft system for a 4-cylinder 4-stroke in line ICE is shown in fig 1.

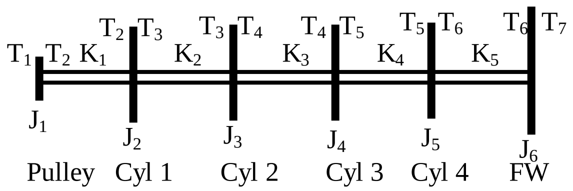

Fig. 1- Crankshaft System of a 4-cylinder ICE

The diagram represents a six degree-of-freedom system. Such a free-free system without external restraining torques, is subjected to dynamic torques as a result of vibratory movement of the masses. This torque must equal the resisting torque due to the maximum twist of the connecting shafts. Hence, the torque *T* needed to vibrate a mass *J* connected by a shaft of stiffness *K* through an amplitude of $\pm a$ radians at a frequency of ω radians/second is given by:

$$T = \pm J w^2 a \quad (1)$$

Assuming a maximum amplitude of $\pm a_1$ radians at mass J_1. Then:

Torque to the left of Mass J_1; is given by

$$T_1 = 0 \quad (2)$$

Torque to the right of Mass J_1;

$$T_2 = T_1 + J_2 w^2 a_1 \quad (3)$$

Twist angle between J_1 and J_2;

$$\frac{T_2}{K_1} = \frac{J_1 w^2 a_1}{K_1} \quad (4)$$

Hence deflection of Mass J_2 is shown in Equation (5) as

$$a_2 = a_1 - \frac{J_1 w^2 a_1}{K_1} \quad (5)$$

Similarly, $T_3 = T_2$+ Torque to accelerate Mass J_2. Hence,

$$T_3 = w^2 a_1 (J_1 + J_2) - \frac{w^4 a_1 J_1 J_2}{K_1} \quad (6)$$

By the same method and reasoning, all the other torques and deflections can be derived, up to the torque (T_7) to the right of the flywheel (Mass J_6) as follows:

$T_7 = \omega^2 a_1[(\sum J_{1\text{-}6}) - \omega^2\{J_1(\sum J_{2\text{-}6})/K_1 + (J_1+J_2) \sum (J_{3\text{-}6})/K_2 + (\sum J(_{1-3}) (\sum J(_{4\text{-}6})/K_3 + (J_5 + J_6)(\sum J_{1-4})/K_4 + J_6(\sum J_{1-5})/K_5\} + \omega_4\{J_1J_2 (\sum J_{3-6})/K_1K_2 + J_1(J_2 + J_3) (\sum J_{4-6})/K_1K_3 + J_3 (J_1 + J_2) (\sum J_{4-6})/K_2K_3 + J_1(\sum J_{2-4}) (J_5 + J_6)/K_1K_4 + (\sum J_{1-2}) (\sum J_{3-4})(\sum J_{5-6})/K_2K_4 + J_4 (\sum J_{1-3})(J_5 + J_6)/K_3K_4 + (\sum J_{1-2}) (\sum J_{3\text{-}4}) (\sum J_{5-6})/K_2K_4 + J_4 (\sum J_{1-3})(J_5 + J_6)/K_3K_4 + J_1J_5J_6/K_1K_5 + J_5J_6 (J_1 + J_2)/K_2K_5 + J_5J_6 (\sum J_{1-3})/K_3K_4 + J_1J_2(\sum J_{3-4}) (\sum J_{5\text{-}6})/K_1K_2K_4 + J_1J_2J_6(\sum J_{3-5}) / K_1K_2K_5 + J_1J_6(\sum J_{2-3})(\sum J_{4-5}) / K_1K_3K_5 + J_5J_6 (\sum J_{1-2})(\sum J_{3\text{-}4})/ K_2K_4K_5 + J_4J_5J_6 (\sum J_{1-3}) / K_3K_4K_5\} + \omega_8\{J_1J_2J_3J_4J_6 (K_4 + K_5)/K_1K_2K_3K_4K_5 + J_3J_5J_6 \{(J_1J_2/K_1)+J_4(J_1 + J_2)/K_4)/K_2K_3K_5\} -\omega^{10}J_1J_2J_3J_4J_5J_6/K_1K_2K_3K_4K_5]$ (7)

Equation (7) can easily be solved using a Holzer Table (Ogbonnaya and Komako, 2006; Nastorides, 2011; Wilson, 2013).

2.1 Torques and Harmonic Components

When the engine is running there is always an instantaneous torque on the crankshaft. This is due to the gas pressure acting on each piston and also the inertia torque due to the reciprocating masses (pistons and connecting rods). As the crankshaft rotates, the direction of the load due to the gas pressure in each cylinder acting on the crank arm varies. Hence, the load at any crank angle θ must be multiplied by the effective crank radius at that angle. The effective crank radius, R_e at crank angle θ is given in Equation (8) (Lilly, 1995):

$$R_e = R\left\{Sin\theta + (Sin2\theta)/2\sqrt{(n^2 - Sin^2\theta)}\right\} \quad (8)$$

Where:
R is the crank radius
n is the connecting rod/crank radius ratio

From a work rate consideration,

Work rate due to torque *T* on crankshaft = *Tω*
Work rate due to gas pressure, p, acting on piston = *pAV*
Hence from a work balance analysis

$$Tw = pAV \quad (9)$$

Where:
A is the piston area
V is the piston speed = *ωR*
Hence,

$$T = pAR$$

Therefore the torque on the crankshaft is

$$T = pAR\left\{Sin\theta + (Sin2\theta)/2\sqrt{(n^2 - Sin^2\theta)}\right\} \quad (10)$$

Thus, an instantaneous gas pressure torque and crank angle (*T–θ*) relationship exists. This relationship produces harmonic sine and cosine components and these vary in magnitude and phase as the crankshaft rotates. A *T–θ* curve can be constructed from this relationship. A Fourier analysis of this relationship will give a constant (*pAR*) and a series of harmonics that vary in amplitude and phase. In each complete engine cycle, these harmonics occur once, twice, thrice etc. Hence in a four-stroke engine the once, twice, thrice etc. harmonics of the engine correspond to the ½, 1, 1½ 2 etc. harmonic orders.

The inertia torque contributes only sine components (Nastorides, 2011). Its magnitude depends on the connecting rod/crank radius ratio, the mass/piston area and the shaft angular acceleration ω^2R (*R* being the crank radius). The resultant excitation torque for any given load is obtained by a combination of the sine components of the inertia and gas torques and the cosine component of the gas torque.

In order to evaluate the sine component of the inertia torque for a given load condition, an inertia coefficient must be first obtained, using such variables as the crank ratio and the engine order number. A table of values of the inertia component of ICEs is available (Nastorides, 2011; Wilson, 2013). The sine component, S_{Ti} is obtained from Equation (11):

$$S_{TI} = T_{NI} m w^2 R \quad (11)$$

Where:
T_{Ni} is the inertia coefficient of the *ith* order
m is the reciprocating mass/piston area ratio
ω is the crankshaft rotational frequency in rad/s
R is the crank radius

Hence, the resultant excitation torque harmonic, *Ti*, of the *ith* order is given by:

$$T_i = \sqrt{\{(S_{TI} + S_{GI})^2 + (C_{GI})^2\}} \quad (12)$$

Where:
S_{Ti} is the sine component of the inertia torque
S_{Gi} is the sine component of the gas torque
C_{Gi} is the cosine component of the gas torque

2. Harmonic Orders

As earlier mentioned, a four-stroke engine exhibits ½, 1, 1½, 2, 2½, 3 etc harmonic orders. The ½, 1½, 2½ etc orders are the half orders. The 1, 3, 5, 7 etc are the minor orders while the 2, 4, 6, 8 etc are the major harmonic orders (Nastorides, 2011; Akhtar, 2009). The amplitudes of vibration of the crankshaft vary according to the harmonic order being exhibited. The variables include:

I. Number of cylinders and configuration.
II. Firing order of the engine.
III. Crankshaft design
IV. Load (torque)

2.2.1 Minor Orders (1, 3, 5 etc)

Consider a 4-cylinder four-stroke inline engine with firing order 1-3-4-2. For the Minor Orders, Cylinder 1 will fire at TDC (0^0), Cylinder 3 will fire 180^0 after TDC, Cylinder 4 will fire 360^0 after TDC while Cylinder 2 will fire 540^0 after Top Dead Centre (TDC). The vector arrangement of the cylinder firing sequence is as shown in Fig. 2. (Nastorides, 2011; Xu and Anderson, 1988).

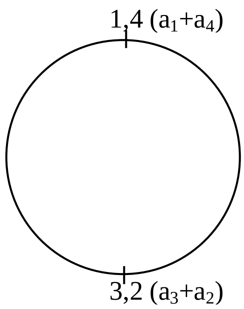

Fig. 2: Vector arrangement for Minor Orders

Assuming a vibration amplitude of magnitude *a* and noting that in the Minor Orders, the torques occur only in the vertical plane, then the vector sum of the amplitude of vibration is a_1Cos 0^0 + a_3 Cos 180^0 + a_4Cos 360^0+a_2Cos 540^0
i.e. $a_1 – a_3 + a_4 – a_2$
Therefore, for this engine, the vector sum for the Minor Orders (1, 3, 5, 7etc) is:

$$\sum a = (a_1 + a_4) - (a_3 + a_2) \qquad (13)$$

2.2.2 Major Orders (2, 4, 6 etc)

For the same engine, the Major Order harmonics occur every 360^0 as the cylinders fire at TDC as shown in Fig. 3 below (Nastorides, 2011; Xu and Anderson, 1988).

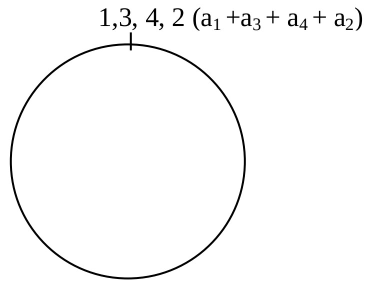

Fig. 3: Vector arrangements for Major Orders

By similar calculations, the vector sum of the amplitudes of vibration for Major Order is:
a_1Cos0^0 + a_3 Cos360^0 + a_4Cos720^0 + a_2Cos1080^0
i.e. $\sum a = a_1 + a_3 + a_4 + a_2$

2.2.3 Half Order (½)

For the same engine, the ½, 2½, 4½, etc. order harmonics occur every 90^0(Nastorides, 2011; Xu and Anderson, 1988). Thus Cylinder 1 fires at 0^0, Cylinder 3 at 90^0, Cylinder 4 at 180^0 and Cylinder 2 at 270^0 (Fig. 4) (Nastorides, 2011; Xu and Anderson, 1988). Here the torquesoccur in both the horizontal and vertical planes. The horizontal components of the amplitudes are:
a_1 Sin 0^0 + a_3 Sin 90^0 + a_4 Sin 180^0 + a_2 Sin 270^0 = $a_3 – a_2$
The vertical components are:
a_1 Cos 0^0 + a_3 Cos 90^0 + a_4 Cos 180^0 + a_2 Cos 270^0 = $a_1 – a_4$
Hence the vector sum $\sum$a for ½ order is:

$$\sum a = \sqrt{\{(a_3 - a_2)^2 + (a_1 - a_4)^2\}} \qquad (14)$$

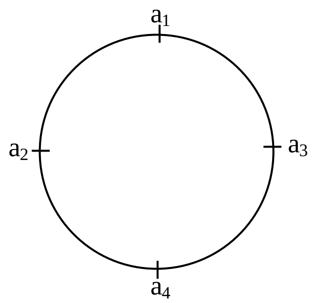

Fig. 4: Vector arrangement for ½, 2½, 4½, etc. orders.

The vectors for the 1½, 3½, 5½, etc orders are spaced 270^0 apart in their firing sequence (Lilly, 1995; Xu and Anderson, 1988) as shown in Fig. 5.

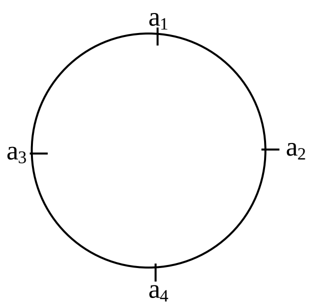

Fig. 5: Vector arrangement for 1½, 3½, 5½, etc. Orders.

In like manner, the horizontal components for these harmonic orders are:

$a_3 \mathrm{Sin}\ 270^0 + a_2\ \mathrm{Sin}\ 540^0$

Also, the vertical components are:

$a_1\ \mathrm{Cos}\ 0^0 + a_4\ \mathrm{Cos}\ 180^0$,

The vector sum $\sum a$ for these harmonic orders is therefore:

$$\sum a = \sqrt{\{(a_2 - a_3)^2 + (a_1 - a_4)^2\}} \quad (15)$$

2.3 Experimental Investigation

In order to ascertain the veracity of the theories elucidated above, some practical investigations were carried out. A Perkins 4 cylinder inline 4 stroke engine was run at constant speed with varying indicated mean effective pressure (imep). The amplitudes of vibration for the different harmonic orders were recorded and captured graphically. The same engine was also run at varying speeds with indicated mean effective pressure kept constant, and the amplitudes of vibration for the various orders also recorded and captured graphically.

The relevant characteristics of the engine are:

Crank ratio	3.45
Piston area	0.00762m^2
Mass/Piston area	350kg/m^2
Crank radius	0.0635m
Firing sequence	1-3-4-2

3.0 Results and Discussion

At constant engine speed, the imep was varied from 4 to10 bar and the vibration amplitudes were recorded for the various harmonic orders as shown in Tables 1 and 2.

Table 1: Variation of vibration amplitude with imep at constant engine speed – Major and Minor Orders

Imep	**Vibration Amplitude**			
(bar)	1^{st} Order	2^{nd} Order	3^{rd} Order	4^{th} Order
4.0	0.003	0.0025	0.0004	0.018
6.0	0.004	0.0045	0.00044	0.0205
8.0	0.005	0.013	0.00046	0.023
10.0	0.006	0.0205	0.00047	0.025

Table 2: Variation of Vibration amplitude with imep at constant engine speed – Half Orders

Imep	**Vibration Amplitude**			
(bar)	½ Order	1½ Order	2½ Order	3½ Order
4.0	0.0265	0.0120	0.0010	0.0070
6.0	0.0290	0.0175	0.0020	0.0080
8.0	0.0330	0.0190	0.0048	0.0090
10.0	0.0370	0.0225	0.0060	0.0100

At constant imep, the engine speed was varied from 150 to 250 rads^{-1} and the vibration amplitudes were recorded for the various harmonic orders as shown in Tables 3 and 4.

Table 3: Variation of Vibration amplitude with engine speed at constant imep – Half Orders

Engine Speed	**Vibration Amplitude**			
(rad $\mathbf{s^{-1}}$)	½ Order	1½ Order	2½ Order	3½ Order
150	0.0320	0.0120	0.0020	0.0050
175	0.0340	0.0095	0.0018	0.0048
200	0.0370	0.0070	0.0060	0.0040
225	0.0420	0.0040	0.0105	0.0035
250	0.0520	0.0030	0.0240	0.0030

Table 4: Variation of Vibration amplitude with engine speed at constant imep – Major and Minor Orders

Engine Speed	**Vibration Amplitude**			
(rad $\mathbf{s^{-1}}$)	1st Order	2^{nd} Order	3rd Order	4th Order
150	0.0045	0.0055	0.0015	0.0200
175	0.0046	0.0065	0.0010	0.0198
200	0.0047	0.0305	0.0005	0.0195
225	0.0048	0.0560	0.0010	0.0150
250	0.0049		0.0015	0.0140

3.1 Effect of Varying Imep at Constant Speed

The results obtained in Table 1 are plotted graphically in Fig. 6. The general trend is that vibration amplitude increases with indicated mean effective pressure for a given order. However, the rate of increase varies from order to order.

3.1.1 Vibration Amplitudes for Minor Orders

The vector sum for the Minor Orders are the same for a given engine. Therefore, their relative amplitudes would be expected to be of about the same magnitude. However, as shown in Fig. 6, this is not so. The differences are due to the torque harmonic components.

The contribution of the inertia torque is constant for a given Minor Order. But the 1^{st} Order inertia sine component is positive while that of the 3^{rd} Order is negative. Since the overall sine component is an addition of the inertia and gas components ($S_{Ti} + SGi$), the negative contribution of the 3^{rd} Order inertia torque reduces the overall sine component for that order. Conversely, the inertia sine contribution of the 1^{st} order is additive, thus the overall sine component of the 1^{st} Order is higher than that of the 3^{rd} Order.

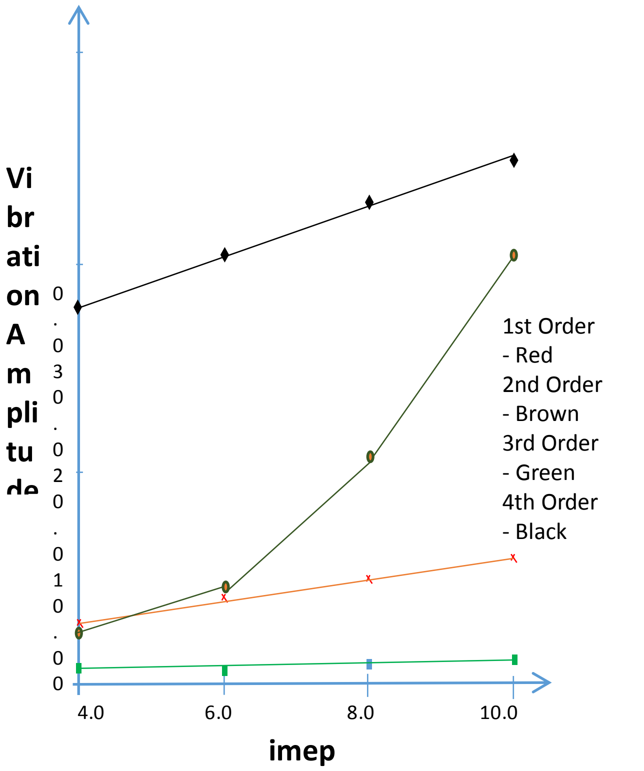

Fig. 6. – Variation of Vibration Amplitude with imep at constant engine speed – Major and Minor Orders

3.1.2 Vibration Amplitudes for Major Orders

As with the Minor Orders the vector sum for the Major Orders are the same (Fig. 3), however, their vibration amplitudes are not the same. The inertia components for the 2nd and 4th Orders are both negative, but the 4th Order component is less negative. Consequently, the subtraction effect of the inertia sine component is higher in the 2nd Order. This, in effect, means that the 4th Order vibration amplitudes are higher than those of the 2nd Order, as shown in Fig. 6.

3.1.3 Vibration Amplitudes for Half Orders

The same phenomenon that applies to the Major and Minor Orders equally applies to the Half Orders, but with some variations. The ½ and 2½ Orders have the same vector sum (Fig 4). The ½ Order inertia sine component is positive while that of the 2½ Order is negative. As a result, the ½ Order amplitudes are higher than those of the 2½ Order (Fig. 7).

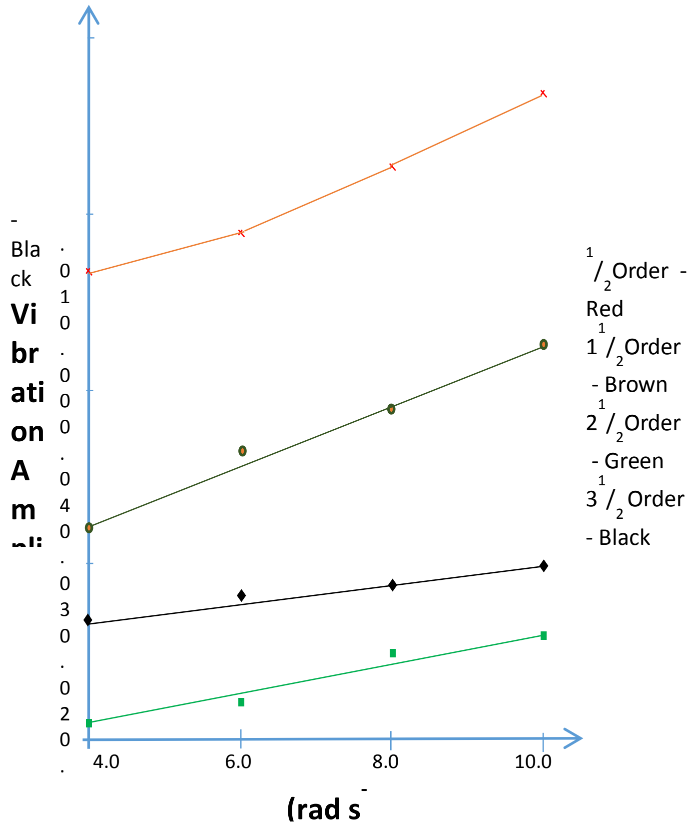

Fig. 7. – Variation of Vibration Amplitude with imep at constant engine speed – Half Orders

The 1½ and 3½ Orders both have the same vector sum (Fig. 4). Their inertia components are both negative, but that of the 3½ Order is more negative. So, as seen in Fig. 7, the 3½ Order amplitudes are lower than the 1½ Order amplitudes.

3.2 Effect of Varying Engine Speed at Constant Imep

When the imep was kept constant and the engine run at various speeds, it was observed that an identical trend as in the previous case manifested. The magnitudes of the amplitudes differed significantly (see Tables 3 and 4). The amplitudes are correspondingly higher when imep is kept constant, as shown in Tables 3 and 4 and in Figs. 8 and 9. This is due to the fact that the inertia component of the harmonic torque plays the dominant role. The gas torques, in this case, are less prominent.

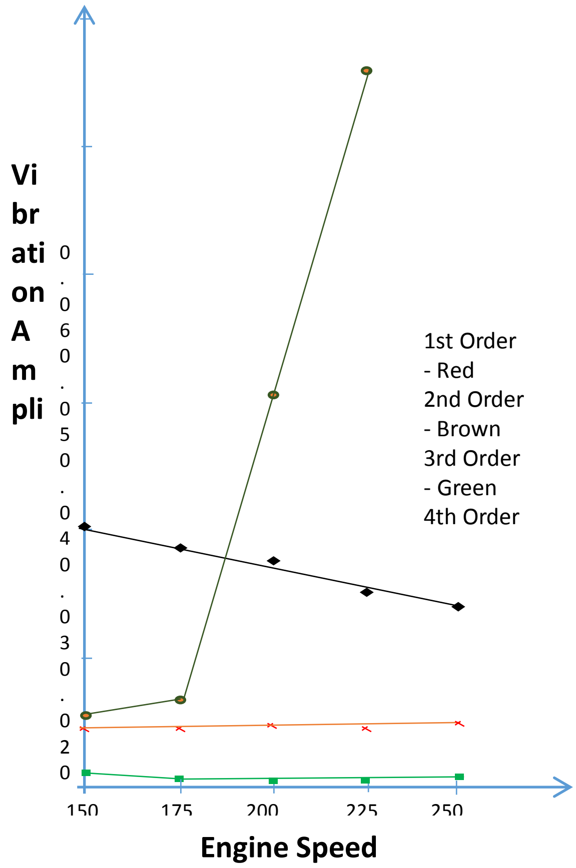

Fig. 8 – Variation of Vibration Amplitude with Engine Speed at constant imep – Half Orders

3.3 Analysis of Condition

Each component of an ICE crankshaft system contributes to the vibration signature through its contribution to the vibration amplitude. The ability to analyze the vibration signatures and with the analysis data make decisions about the condition of the engine and therefore detect incipient or potential failures is vital in using crankshaft torsional vibration as a tool for condition monitoring. One level of interpretation is quantitative as it is dependent on analyses of the vibration amplitudes at various specific frequencies. This finds common use in ICEs employed as propulsion engines where the speeds vary. This level of interpretation gives more definite indications of faults and impending failures in the system.

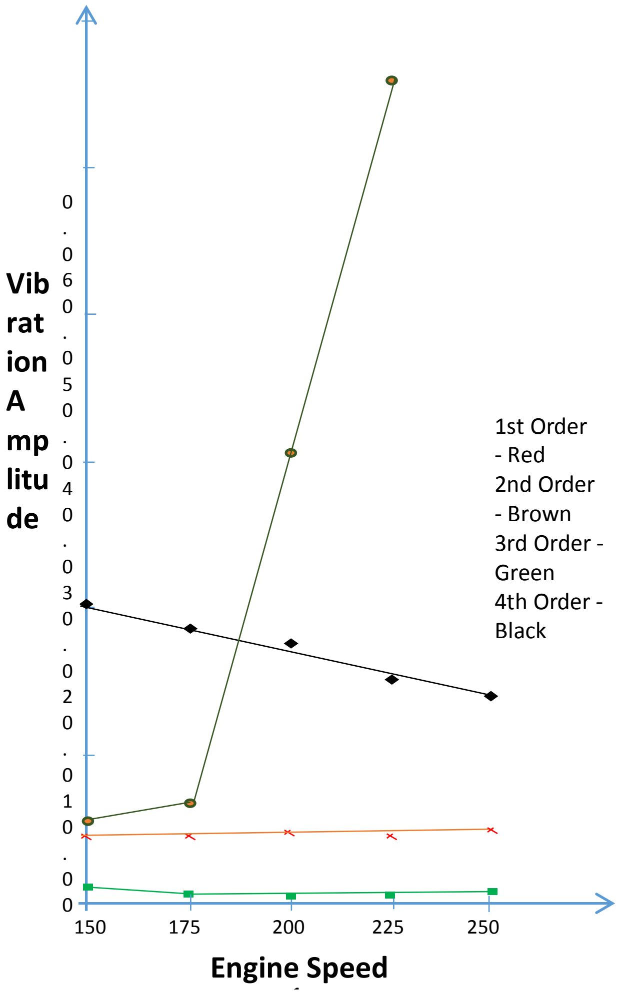

Fig. 9 – Variation of Vibration Amplitude with Engine Speed at constant imep – Major and Minor Orders

3.4 Analysis of Vibration Amplitudes

A good analysis of the crankshaft torsional vibration amplitudes leads to detection on specific defects, if any, in the engine. It therefore suggests that a sound understanding of the characteristics of vibration amplitudes is essential for its use in condition monitoring of the engine. As an example, in the four-stroke engine, there is a firing pulse in every two complete revolutions of the crankshaft. The fundamental firing frequency is therefore a half order phenomenon. Therefore, harmonics of the half order should be present in the crankshaft vibration amplitudes of a four-stroke engine. By inference, therefore, absence of half order harmonic contents in vibration amplitude indicates incomplete firing sequence.

In this work, two specific aspects of the condition monitoring of an ICE will be examined: cylinder firing sequence and cylinder conditions.

3.5 Cylinder Firing Sequence

In an even firing two cylinder four-stroke engine the first cylinder fires at TDC (or 0^0), the second fires 360^0

later and the first refires 720^0 from TDC. Similarly, in a four cylinder four-stroke engine, the first cylinder fires at TDC, the second firing cylinder fires 180^0 later, the third 360^0, the fourth 540^0 and the first refires 720^0 after TDC. The cylinder firing sequence for a given engine is thus constrained mechanically by the cylinder orientation and the relative crankpin spacing. This relative phasing between cylinders affects crankshaft torsional vibration. Evenly firing two cylinder four-stroke engines do not exhibit a 1½ order torsional vibration, whereas four-stroke four cylinder engines do. If the 1½ order amplitude of a four cylinder engine is zero, then the engine is behaving like a two cylinder engine, suggesting that two of the cylinders are not firing.

By similar analyses, a four cylinder four-stroke engine firing evenly exhibits second, fourth and sixth order vibration amplitudes of nearly equal magnitude. The amplitudes of the Minor Orders will be relatively less than those of the Major Orders.

3.6 Cylinder Conditions

The effects on torsional vibration of irregular or non-periodic events in engine operation may be deduced from the cylinder phasing considerations. These effects are indicative of the cylinder conditions. If one or more cylinders are not firing properly, it will affect the vibration of the engine and will become apparent in the torsional vibration signatures. Analysis of the vibration amplitudes of various order harmonics will indicate which cylinders are affected.

Cylinder wear can also be detected from the vibration amplitudes. Since the deflection or vibration amplitude of each cylinder is proportional to the excitation torque and the torque is proportional to the cylinder gas pressure, a relatively low amplitude in any cylinder indicates a gas pressure reduction in the cylinder. This can be due to cylinder liner wear or malfunction of the rings. This can be confirmed with a trend analysis.

3.7 **Validation**

The results obtained from this work is validated using those contained in Ogbonnaya (1998) and Ogbonnaya (2004) where torsional vibration analysis were carried out using an MTU 12V 396TC 32 and an H26 Gas Turbine Generator respectively. The two works cited above also show that the amplitude of vibration of the rotating system vary in magnitude according to the condition of the engines.

4. Conclusion

When an Internal Combustion Engine (ICE) is running, its crankshaft is subjected to torsional vibrations. The excitation torques on the crankshaft are produced by gas pressure in each cylinder and inertia torques caused by the reciprocating masses (pistons and connecting rods). Cylinder gas pressure varies with crank angle, resulting in harmonic sine and cosine components. This harmonic components manifest in harmonic orders (Half, Major and Minor Orders). The inertia torque contributes only sine components while the gas torque contributes both sine and cosine components.
The amplitudes of vibration of the crankshaft system vary in magnitude according to the condition of the engine. Considering specifically the cylinder firing sequence and the cylinder conditions, it has been established that the vibration amplitudes of the crankshaft system can be used to determine the condition of the engine. To do this, a trend analysis of the vibration amplitudes is carried out and for known operating conditions (engine speed and indicated mean effective pressure) marked deviations of vibration amplitudes indicate incipient faults and potential failures. The firing characteristics of the cylinders can be judged, as well as the conditions of the cylinders. Early detection of incipient faults and determination of remedial actions contribute to preventive maintenance which increases the reliability and hence availability of the engine.
The contribution of the inertia torques and gas torques on a crankshaft system of an ICE determine the magnitude of the amplitudes of vibration. Thus, a careful analysis of the effects of the inertia and gas torques on the crankshaft system can contribute to preventive maintenance of the engine.

REFERENCES

Akhtar, (2009).*Harmonics in 4-cylinder engines*; Contributions on the internet to issues raised *www.sect\shareddocs\harmonicsin4cylinderengines*: posted 24 August 2009.

Lilly L. C. R., (1995). *Diesel Engine Reference Book, Butterworth*; Economic Zone Catalogue, 2nd edition.

Meier H., Peter and Bernhardt F., (2009). *Compendium Marine Engineering*, DW Media Group, Seehafen Verlag, Hamburg; 1st Edition.

Nestorides E. J., (2011). *BICERA – A Handbook of Torsional Vibrations*, Cambridge University Press.

Ogbonnaya, EA (1998) *Condition Monitoring of a Diesel Engine for Electricity Generation*. MTech Thesis, Rivers State University of Science and Technology (RSUST), Port Harcourt-Nigeria. pp 89-97

Ogbonnaya, EA (2004) *Modelling Vibration-Based Faults in Rotor Shaft of a Gas Turbine*, PhD Thesis, RSUST, Port Harcourt-Nigeria. pp 155-160

Ogbonnaya, EA and Komako, KE (2006) Adoption of Holzer Torsional Analysis to Diesel Engine Electricity Generator, *Journal of Agriculture and Environment Engineering Technology*, Vol. 2, No.1 (www.jaeetng.net)

Ogbonnaya, EA (2006) Application of Torsional Vibration Analysis in System Maintenance, *Academy Journal of Science and Engineering*. Vol. 4, No.1, pp. 46-50.

Pounder C. C., Wilbur C.T. and Wight D. A., (1991).*Marine Diesel Engines*, Butterworth Heinemann Ltd.

Wilson W. K.,(2013) *Practical Solution of Torsional Vibration Problems*, Vol. II', Chapman and Hall, reprinted edition.

Xu Z. and Anderson R. J., (1988)*A New Method For Estimating amplitudes of Torsional Vibration for Engine Crankshafts*; *International Journal of Vehicle Design*, Vol. 9 No. 2